01

The set theory was developed by German mathematician Georg Cantor (1845-1918). He first encountered sets while working on 'problems on trigonometric series'. This concept is used in every branch of Mathematics like Geometry, Algebra, etc. Further, sets are used to define relations and functions.

SETS

CHAPTER CHECKLIST

- Sets and Their Representations
- Types of Sets
- Subset and Universal Set
- Venn Diagrams and Operations on Sets
- Applications of Set Theory

|TOPIC 1|

Sets and Their Representations

In our daily life, while performing regular work, we often come across a variety of things that occur in groups. e.g. Team of cricket players, group of tall boys, a pack of cards, etc.

The words used above like team, group, pack, etc., convey the idea of certain collections.

WELL-DEFINED COLLECTION OF OBJECTS

If any given collection of objects is in such a way that it is possible to tell without any doubt whether a given object belongs to this collection or not, then such a collection of objects is called a **well-defined collection of objects**.

e.g. 'The rivers of India' is a well-defined collection. Since, we can say that the river Nile does not belong to this collection. On the other hand, the river Ganga belongs to this collection.

Difference between Not Well-defined and Well-defined Collections

Not well-defined collection	Well-defined collection
A group of intelligent students.	A group of students scoring more than 95% marks of your class.
A group of most talented writers of India.	A group of odd natural numbers less than 25.
Group of pretty girls.	Group of girls of class XI of your school.

Here, a group of intelligent students, a group of most talented writers of India, group of pretty girls are not well-defined collections, because the term intelligent, talented, pretty are vagu and the criterian for determining intelligent students, talented writers and pretty girls may vary from person-to-person.

DEFINITION OF SET

A **well-defined** collection of objects, is called a set. Sets are usually denoted by the capital letters A, B, C, X, Y, Z etc. The elements of a set are represented by small letters a, b, c, x, y, z etc.

If a is an element of a set A, then we say that a belongs to A. The word *'belongs to'* denoted by the Greek symbol $\in$ (epsilon).

Thus, in notation form, a belongs to set A is written as $a \in A$ and b does not belong to set A is written as $b \notin A$.

e.g. (i) If $A = \{1, 2, 3, 4, 5\}$, then $3 \in A$ and $6 \notin A$.

(ii) If P being the set of perfect square numbers, then $36 \in P$ but $5 \notin P$.

Note Objects, elements and members of a set are synonymous terms.

EXAMPLE |1| Which of the following are sets? Justify your answer. **[NCERT]**

(i) The collection of all the months of a year beginning with the letter J.

(ii) The collection of ten most talented writers of India.

(iii) A collection of novels written by the writer Munshi Prem Chand.

(iv) A collection of most dangerous animals of the world.

Sol. (i) We are sure that members of this collection are January, June and July.

So, this collection is well-defined. Hence, it is a set.

(ii) A writer may be most talented for one person and may not be for other. Therefore, we cannot definitely decide which writer will be there in the collection.

So, this collection is not well-defined. Hence, it is not a set.

(iii) Here, we can definitely decide whether a given novel is written by Munsi Prem Chand or not

So, this collection is well-defined. Hence, it is a set.

(iv) The term most dangerous is vague term. An animal may be most dangerous for one person and may not be for the other.

So, it is not well-defined. Hence, it is not a set.

EXAMPLE |2| If $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then insert appropriate symbol $\in$ or $\notin$ in each of the following blank spaces.

(i) $4 \ldots A$ (ii) $-4 \ldots A$ (iii) $12 \ldots A$

(iv) $9 \ldots A$ (v) $0 \ldots A$ (vi) $-2 \ldots A$

Sol. Given, $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(i) Since, 4 is an element of A, therefore $4 \in A$.

(ii) Since, -4 is not an element of A, therefore $-4 \notin A$.

(iii) Since, 12 is not an element of A, therefore $12 \notin A$.

(iv) Since, 9 is an element of A, therefore $9 \in A$.

(v) Since, 0 is an element of A, therefore $0 \in A$.

(vi) Since, -2 does not an element of A, therefore $-2 \notin A$.

Some examples of sets used particularly in Mathematics

$N \rightarrow$ The set of natural numbers.

$Z \rightarrow$ The set of all integers.

$Q \rightarrow$ The set of all rational numbers.

$R \rightarrow$ The set of real numbers (rational and irrational numbers).

$Z^+ \rightarrow$ The set of positive integers.

$Q^+ \rightarrow$ The set of positive rational numbers.

$R^+ \rightarrow$ The set of positive real numbers (positive rational and numbers)

Note From now onwards, we use the symbols $N, Z, Q, R, Z^+, Q^+, R^+$ for the above sets only.

REPRESENTATIONS OF SETS

Sets are generally represented by following two ways

1. Roster Form or Tabular Form or Listing Method
2. Set-Builder Form or Rule Method

Roster form or Tabular Form or Listing Method

In this form, all the elements of a set are listed, the elements are being separated by commas and are enclosed within curly braces { }.

e.g.

(i) The set of all natural numbers less than 10 is represented in roster form as $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

(ii) The set of prime numbers is $\{2, 3, 5, 7, \ldots\}$. Here, three dots tell us that the list of prime numbers continue indefinitely.

Note

(i) In roster form, order in which the elements are listed is not important. i.e. the set of natural numbers less than 10 can also be written as $\{2, 4, 1, 3, 5, 6, 8, 7, 9\}$ instead of $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

(ii) In roster form, element is not repeated, i.e. all the elements are taken as distinct. e.g. The set of letters forming the word 'MISCELLANEOUS' is {M, I, S, C, E, L, A, N, O, U}.

EXAMPLE |3| Write the set of all vowels in English alphabet which precedes 's'.

Sol. The vowels which precedes 's' are a, e, i and o.

So, the required set is $A = \{a, e, i, o\}$.

EXAMPLE |4| Describe the following set in Roster form. $\{x : x$ is positive integer and a divisor of $9\}$

Sol. Here, x is a positive integer and a divisor of 9.

So, x can take values 1, 3, 9.

$\therefore$ $\{x : x$ is a positive integer and a divisor of $9\} = \{1, 3, 9\}$

EXAMPLE |5| Write the set of all natural numbers x such that $4x + 9 < 50$ in roster form.

Firstly, simplify the inequality and then list all the natural numbers under given condition.

Sol. We have, $4x + 9 < 50$

$\Rightarrow$ $4x + 9 - 9 < 50 - 9$ [subtracting 9 from both sides]

$\Rightarrow$ $4x < 41 \Rightarrow x < \frac{41}{4}$

$\therefore$ $x < 10.25$

Since, x is a natural number, so x can take values 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

$\therefore$ Required set $= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

EXAMPLE |6| List the elements of the following sets.

(i) $\{x : x \in Z$ and $|x| \le 2\}$

(ii) $\{x : x = \frac{n}{1+n^2}$ and $1 \le n \le 3$, where $n \in N\}$

Sol. (i) Given, $\{x : x \in Z$ and $|x| \le 2\}$

Since, $|x| \le 2$

$\therefore$ $-2 \le x \le 2$

Also, as $x \in Z$, therefore $x = -2, -1, 0, 1, 2$

Hence $\{x : x \in Z$ and $|x| \le 2\} = \{-2, -1, 0, 1, 2\}$

(ii) Given, $\{x : x = \frac{n}{1+n^2}$ and $1 \le n \le 3$, where $n \in N\}$

Here, $x = \frac{n}{1+n^2}, 1 \le n \le 3, n \in N$

$\Rightarrow$ $x = \frac{n}{1+n^2}, n = 1, 2, 3$

$\Rightarrow$ $x = \frac{1}{1+1^2}, \frac{2}{1+2^2}, \frac{3}{1+3^2} = \frac{1}{2}, \frac{2}{5}, \frac{3}{10}$

$\therefore$ Required set is $\left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}\right\}$.

EXAMPLE |7| Use listing method to express the following sets.

(i) $A = \{a_n : n \in N, a_{n+1} = 3a_n$ and $a_1 = 1\}$

(ii) $B = \{a_n : n \in N, a_{n+2} = a_{n+1} + a_n$ and $a_1 = a_2 = 1\}$

Sol. (i) We have, $a_1 = 1$ and $a_{n+1} = 3a_n, \forall n \in N$

On putting $n = 1, 2, 3, 4, \dots$ in $a_{n+1} = 3a_n$, we get

$$a_2 = 3a_1 = 3 \times 1 = 3$$
$$a_3 = 3a_2 = 3 \times 3 = 3^2$$
$$a_4 = 3a_3 = 3 \times 3^2 = 3^3$$
$$a_5 = 3a_4 = 3 \times 3^3 = 3^4$$
$$a_6 = 3a_5 = 3 \times 3^4 = 3^5 \text{ and so on.}$$

$\therefore$ $A = \{1, 3, 3^2, 3^3, 3^4, 3^5, \dots\}$

(ii) We have, $a_1 = 1, a_2 = 1$

and $a_{n+2} = a_{n+1} + a_n, \forall n \in N.$

On putting $n = 1, 2, 3, \dots$ in $a_{n+2} = a_{n+1} + a_n$, we get

$$a_3 = a_2 + a_1 = 1 + 1 = 2$$
$$a_4 = a_3 + a_2 = 2 + 1 = 3$$
$$a_5 = a_4 + a_3 = 3 + 2 = 5$$
$$a_6 = a_5 + a_4 = 5 + 3 = 8 \text{ and so on.}$$

Note $B = \{1, 1, 2, 3, 5, 8, \dots\}$

Set-builder Form or Rule Method

In this form, all the elements of set possess a single common property $p(x)$, which is not possessed by any other element outside the set.

In such a case, the set is described by $\{x : p(x)$ holds$\}$.

e.g. In the set $\{a, e, i, o, u\}$, all the elements possess a common property, namely, each of them is a vowel of English alphabet. Note that no other letter possesses this property.

If we denote the set of vowels by V, then we write

$$V = \{x : x \text{ is a vowel in English alphabet}\}$$

The above description of the set V is read as

The set of all x such that x is a vowel in English alphabet.

Some examples are given below

(i) Set of all natural numbers less than 10,

$$A = \{x : x \in N \text{ and } x < 10\}$$

(ii) Set of all real numbers,

$$B = \{x : x \in R\}$$

WORKING RULE TO WRITE THE SET IN THE SET-BUILDER FORM

To convert the given set in set-builder form, *we use the following steps*

***Step* I** Describe the elements of the set by using a symbol x or any other symbol y, z etc.

***Step* II** Write the symbol colon ':'.

***Step* III** After the sign of colon, write the characteristic property possessed by the elements of the set.

***Step* IV** Enclose the whole description within curly braces i.e. { }.

EXAMPLE |8| Write the set $A = \{14, 21, 28, 35, 42, ..., 98\}$ in set-builder form.

Firstly, find the characteristic property possessed by the elements of the set, then write in set-builder form.

Sol. Let x represents the elements of given set.

Given numbers are natural numbers greater than 13, less than 99 and multiples of 7.

Thus, $A = \{x : x$ is a natural number greater than 13, less than 99 and a multiple of 7$\}$, which is the required set-builder form of given set.

This can also be written as

$A = \{x : x$ is a natural number, a multiple of 7 and $13 < x < 99\}$

or $A = \{x : x = 7n, n \in N$ and $2 \le n \le 14\}$

EXAMPLE |9| Describe the following set in set-builder form $B = \{53, 59, 61, 67, 71, 73, 79, 83, 89, 97\}$

Sol. The given set is $\{53, 59, 61, 67, 71, 73, 79, 83, 89, 97\}$.

We observe that these numbers are all prime numbers between 50 and 100.

$\therefore$ Given set,

$B = \{x : x$ is a prime number and $50 < x < 100\}$

EXAMPLE |10| Write the set of all real numbers which cannot be written as the quotient of two integers in the set-builder form.

Sol. We know that all rational numbers are expressible as the quotient of two integers. Therefore, the required set is the set of those real numbers which are irrational.

$\therefore$ Required set is $\{x : x \in R$ but $x \notin Q\}$.

Representation of a Statement in Both Form

Statement	Roster form	Set-builder form
The set of all natural numbers between 10 and 14.	$\{11, 12, 13\}$	$\{x : x \in N$ and $10 < x < 14\}$
The set of all prime numbers less than 11.	$\{2, 3, 5, 7\}$	$\{x : x$ is a prime number and $x < 11\}$
The set of all distinct letters used in the word 'Friend'.	{F, r, i, e, n, d}	$\{x : x$ is a letter used in the word 'Friend'$\}$

EXAMPLE |11| Describe the following.

(i) "The set of all vowels in the word EQUATION" in roster form.

(ii) "The set of reciprocals of natural numbers" in set-builder form.

Sol. (i) The word "EQUATION" has following vowels, i.e. A, E, I, O and U.

Now, the given statement can be described in roster form as {A, E, I, O, U}.

(ii) Given statement can be described in set-builder form as $\{x : x$ is a reciprocal of a natural number$\}$

or $\left\{x : x = \frac{1}{n}, n \in N\right\}$

TOPIC PRACTICE 1

OBJECTIVE TYPE QUESTIONS

1 Which of the following is not a set?
(a) The collection of days of week.
(b) The collection of prime number less than 10.
(c) The collection of intelligent students in your class.
(d) All of the above.

2 If b is not an element of A, then we write
(a) $b \in A$ (b) $A \in b$ (c) $b \notin A$ (d) $A \notin b$

3 The set of natural numbers which divide 42 is
(a) {1, 2, 3} (b) {1, 2, 3, 6, 7, 14, 21}
(c) {1, 2, 3, 6, 7, 14, 21, 42} (d) {42}

4 The set of all letters in the word 'BETTER' in roster form is
(a) {B, E, T, R} (b) {B, T, R}
(c) {B, E, R} (d) {B, R}

5 The set $A = \{x : x$ is an odd natural number$\}$ in the roster form is ...X... . Here, X refers to
(a) {1, 3, 5, 7} (b) {1, 3, 5, 7, ...}
(c) {3, 5, 7, ...} (d) None of these

VERY SHORT ANSWER Type Questions

Directions (Q. Nos. 6-8) *Which of the following are sets? Justify your answer.*

6 A team of eleven best cricket batsman of the world. **[NCERT]**

7 The collection of all boys in your class. **[NCERT]**

8 The collection of all natural numbers less than 100.

9 If $A = \{1, 3, 5, 7, 9, 11, 13, 15\}$, then insert the appropriate symbol $\in$ or $\notin$ in each of the
(i) 1 ... A (ii) 6 ... A (iii) 9 ... A (iv) 14 ... A

10 Write the set $A = \{x : x \in Z, x^2 < 20\}$ in the roster form.

11 Write $\{x : x$ is an integer and $-3 \le x < 7\}$ in roster form. **[NCERT]**

12 Write set $A = \{3, 6, 9, 12, 15\}$ in set-builder form.

13 Write set $A = \{1, 4, 9, ..., 100\}$ in set-builder form. **[NCERT]**

14 Express the set

$D = \left\{x : x = \dfrac{n^2 - 1}{n^2 + 1}, n \in N \text{ and } n < 4\right\}$ in roster form.

15 Describe the following.

(i) "The set of vowels in the word MATHEMATICS" in roster form.

(ii) "The set of all odd natural numbers" in set-builder form.

16 Describe the following set in roster form.

$\{x : x$ is a letter of the word PROPORTION$\}$

SHORT ANSWER Type Questions

17 Let $A = \{1, 2, 3, 4\}$, $B = \{5, 6, 7\}$ and $C = \{7, 8, 9, 10\}$. Insert the correct symbol $\in$ or $\notin$ in each of the following blanks.

(i) $2 \ldots A$ (ii) $5 \ldots C$

(iii) $6 \ldots B$ (iv) $7 \ldots A$

(v) $5 \ldots B$ (vi) $10 \ldots C$

18 Describe the following sets in roster form.

(i) The set of all letters in the word 'ALGEBRA'.

(ii) The set of all natural numbers less than 7.

(iii) The set of squares of integers.

(iv) The set of all letters in the word "TRIGONOMETRY".

19 Describe the following sets in set-builder form.

(i) The set of all letters in the word PROBABILITY.

(ii) The set of all even natural numbers.

(iii) $\{5, 25, 125, 625\}$

HINTS & ANSWERS

1 (c) The collection of intelligent students in your class is not set.

2 (c) If 'b' is not an element of a set A, then we write $b \notin A$ and read 'b does not belong to A'.

3 (c) Factors of 42 are 1, 2, 3, 6, 7, 14, 21, 42.
The set of natural numbers which divide 42, is represented by $\{1, 2, 3, 6, 7, 14, 21, 42\}$.

4 (a) There are 6 letters in the word 'BETTER' and two letters E and T are repeated. So, the given set can be represented in roster form as {B, E, T, R}.

5 (b) The given set can be represent in roster form as $\{1, 3, 5, 7, \ldots\}$.

6 Not a set

7 Set

8 Set

9 (i) $1 \in A$

(ii) $6 \notin A$

(iii) $9 \in A$

(iv) $14 \notin A$

10 It can be seen that the square of integers $0, \pm 1, \pm 2, \pm 3, \pm 4$ are less than 20.
Ans. $A = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

11 $\{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$

12 $A = \{x : x$ is a natural number multiple of 3 and $x < 18\}$

13 $A = \{x : x = n^2, n \in N$ and $n < 11\}$

14 $D = \left\{0, \dfrac{3}{5}, \dfrac{4}{5}\right\}$

15 (i) The word "MATHEMATICS" has following vowels i.e. A, E and I.
Ans. {A, E, I}.

(ii) An odd natural number can be written in the form $(2n - 1)$. Given set can be described in set-builder form
Ans. $\{x : x = 2n - 1, n \in N\}$

16 Distinct letters in the word PROPORTION are P, R, O, T, I, N. So, x can be P, R, O, T, I, N.
Ans. Required set = {P, R, O, T, I, N}

17 (i) Since, 2 is an element of A. So, $2 \in A$

(ii) Since, 5 is not an element of A. So, $5 \notin C$

(iii) $6 \in B$ (iv) $7 \notin A$

(v) $5 \in B$ (vi) $10 \in C$

18. (i) {A, L, G, E, B, R}

(ii) $\{1, 2, 3, 4, 5, 6\}$

(iii) $\{0, 1, 4, 9, 16, \ldots\}$

(iv) {T, R, I, G, O, N, M, E, Y}

19. (i) $\{x$: x is a letter of the word PROBABILITY$\}$

(ii) $\{x : x = 2n, n \in N\}$ or $\{2n : n \in N\}$

(iii) $\{x : x = 5^n, n \in N$ and $n < 5\}$
or $\{5^n : n \in N$ and $n < 5\}$

|TOPIC 2|

Types of Sets

In this topic, we will study about different types of sets and cardinal number of a set.

Empty Set

A set which does not contain any element, is called an empty set or null set or void set. It is denoted by ϕ or $\{\ \}$.

e.g. $A = \{x : x$ is a natural number less than $1\}$

We know that, there is no natural number less than one. Therefore, set A contains no element and hence it is an empty set.

EXAMPLE |1| Which of the following sets are empty sets?

(i) Set of all even natural numbers divisible by 5.

(ii) Set of all even prime numbers. **[NCERT]**

(iii) $\{x : x^2 - 2 = 0$ and x is rational$\}$

(iv) $\{x : x$ is a point common to any two parallel lines$\}$ **[NCERT]**

Sol. (i) There are infinite even natural numbers which are divisible by 5. e.g. 10, 20, 30, 40, 50, etc.

Therefore, it is not an empty set.

(ii) We know that 2 is only even prime.

Therefore, it is not an empty set.

(iii) Given, $\{x : x^2 - 2 = 0$ and x is rational$\}$

Here, $x^2 - 2 = 0 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2} \notin Q$

Therefore, it is an empty set.

(iv) Given, $\{x : x$ is a point common to any two parallel lines$\}$

We know that, no point is common to any two parallel lines. Therefore, it is an empty set.

EXAMPLE |2| Which of the following sets are empty?

(i) $A = \{x : x \in N$ and $x \le 1\}$

(ii) $B = \{x : 3x + 1 = 0, x \in N\}$

(iii) $C = \{x : 2 < x < 3, x \in N\}$

Sol. (i) We have, $A = \{x : x \in N$ and $x \le 1\}$

$\Rightarrow \quad A = \{1\}$

So, this is not an empty set.

(ii) We have, $B = \{x : 3x + 1 = 0, x \in N\}$

Since, $3x + 1 = 0$

$\Rightarrow \quad x = \frac{-1}{3} \notin N$

Thus, set B does not contain any element.

Hence, set B is an empty set.

(iii) We have, $C = \{x : 2 < x < 3, x \in N\}$

Since, there is no natural number between 2 and 3.

Hence, set C is an empty set.

Singleton Set

A set, consisting of a single element, is called a singleton set.

e.g.

(i) The sets $\{0\}, \{5\}, \{-7\}$ are singleton sets.

(ii) $A = \{x : x + 8 = 0, x \in Z\}$ is a singleton set, because this set contains only one integer, namely -8.

EXAMPLE |3| Let $T = \left\{x : \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x}\right\}$.

Is T a singleton set? Justify your answer.

Sol. We have, $T = \left\{x : \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x}\right\}$

$$\because \quad \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x} \Rightarrow \frac{x+5}{x-7} - \frac{5}{1} = \frac{4x-40}{13-x}$$

$$\Rightarrow \frac{x+5-5x+35}{x-7} = \frac{4x-40}{13-x} \Rightarrow \frac{40-4x}{x-7} = \frac{4x-40}{13-x}$$

$$\Rightarrow \frac{-(4x-40)}{x-7} - \frac{4x-40}{13-x} = 0$$

$$\Rightarrow -(4x-40)\left[\frac{1}{(13-x)} + \frac{1}{(x-7)}\right] = 0$$

$$\Rightarrow (4x-40)\left[\frac{6}{(13-x)(x-7)}\right] = 0 \Rightarrow 4x - 40 = 0$$

$$\therefore \quad x = 10$$

Hence, T is a singleton set.

Finite Set

A set which is empty or consists of a definite number of elements, is called a finite set.

e.g.

(i) The set $\{1, 2, 3, 4\}$ is a finite set, because it contains a definite number of elements, i.e. only 4 elements.

(ii) The set of solutions of $x^2 = 25$ is a finite set, because it contains a definite number of elements, namely 5 and -5.

CARDINAL NUMBER

The number of distinct elements in a finite set A is called **cardinal number** of set A and it is denoted by $n(A)$.

e.g. If $A = \{-3, -1, 8, 10, 13\}$, then $n(A) = 5$.

Infinite Set

A set which consists of infinite number of elements is called an infinite set.

When a set is infinite set, it is not possible to write all the elements within braces { } because the number of elements of such a set is not finite. In such cases, we write a few elements which clearly indicate the structure of the set followed by three dots.

e.g. Set of squares of natural numbers is an infinite set, because such natural numbers are infinite and it can be represented in roster form as $\{1, 4, 9, 16, 25, \ldots\}$.

Note All infinite sets cannot be described in the roster form. e.g. The set of real numbers cannot be described in this form, because the elements of this set does not follow any particular pattern.

EXAMPLE |4| Which of the following sets are finite and which are infinite?

(i) Set of concentric circles in a plane.

(ii) The set of lines which are parallel to the X-axis.

(iii) The set of animals living on the Earth. **[NCERT]**

(iv) The set of circles passing through the origin (0, 0). **[NCERT]**

Sol. (i) We can draw infinite circles having same centre.

$\therefore$ It is an infinite set.

(ii) We can draw infinite lines parallel to X-axis.

$\therefore$ It is an infinite set.

(iii) There are finite number of animals living on Earth.

$\therefore$ It is a finite set.

(iv) We can draw infinite number of circles, passing through origin.

$\therefore$ It is an infinite set.

EXAMPLE |5| Which of the following sets are finite and which are infinite?

(i) $\{x \in R : 0 < x < 1\}$ (ii) $\{x \in Z : x < 5\}$

(iii) The set of positive integers greater than 100. **[NCERT]**

(iv) The set of prime numbers less than 99.

Sol. (i) Given, $\{x \in R : 0 < x < 1\}$

Here, $0 < x < 1$

We know that between any two real numbers, there are infinite real numbers.

$\therefore$ The set $\{x \in R : 0 < x < 1\}$ is an infinite set.

(ii) Given, $\{x \in Z : x < 5\}$. Here, $x < 5$

We know that, there are infinite integers less than 5. i.e. 4, 3, 2, 1, 0, -1, etc.

$\therefore$ The set $\{x \in Z : x < 5\}$ is an infinite set.

(iii) It is an infinite set because there are infinite numbers greater than 100, i.e. 101, 102, 103, 104, etc.

(iv) If is a finite set because there are 25 prime numbers less than 99, namely 2, 3, 5, 7, 11, ..., 97.

Equivalent Sets

Two sets A and B are said to be equivalent, if their **cardinal numbers** are same i.e. if $n(A) = n(B)$.

e.g. Let $A = \{a, b, c, d\}$ and $B = \{1, 2, 3, 4\}$, then $n(A) = 4$ and $n(B) = 4$.

Therefore, A and B are equivalent sets.

EXAMPLE |6| From the following sets given below, pair the equivalent sets.

$A = \{1, 2, 3\}$, $B = \{t, p, q, r, s\}$, $C = \{\alpha, \beta, \gamma\}$ and $D = \{a, e, i, o, u\}$

Sol. Given, $A = \{1, 2, 3\} \Rightarrow n(A) = 3$

$B = \{t, p, q, r, s\} \Rightarrow n(B) = 5$

$C = \{\alpha, \beta, \gamma\} \Rightarrow n(C) = 3$

$D = \{a, e, i, o, u\} \Rightarrow n(D) = 5$

Here $n(A) = n(C) = 3$ and $n(B) = n(D) = 5$

$\therefore$ The sets A, C and B, D are equivalent sets.

Equal Sets

Two sets A and B are said to be equal, if they have exactly the same elements and we write it as $A = B$. Otherwise, two sets are said to be **unequal** and we write $A \neq B$.

e.g. Let $A = \{a, b, c, d\}$ and $B = \{c, d, b, a\}$, then $A = B$, because each element of A is in B and *vice-versa*.

Note A set does not change, if one or more elements of the set are repeated.
e.g. The sets $A = \{1, 4, 5\}$ and $B = \{1, 1, 4, 5, 5\}$ are equal because elements of A is in B and *vice-versa*. That's why, we generally do not repeat any element in describing a set.

EXAMPLE |7| From the following sets, select equal sets.

$A = \{2, 4, 6, 8\}$, $B = \{1, 2, 3, 4, 5\}$,
$C = \{-2, 4, 6, 8\}$, $D = \{2, 3, 5, 4, 1\}$,
$E = \{8, 6, 2, 4\}$

Sol. Since, each element of set A presents in set E and *vice-versa*.

Therefore, A and E are equal sets, i.e. $A = E$.

Also, each element of set B presents in set D and *vice-versa*.

Therefore, B and D are equal sets.

EXAMPLE |8| Are the following pair of sets equal? Give reason. **[NCERT]**

(i) $A = \{2, 3\}$ and $B = \{x : x \text{ is a solution of } x^2 + 5x + 6 = 0\}$

(ii) $A = \{x : x \text{ is a letter of the word "FOLLOW"}\}$
and $B = \{y : y \text{ is a letter of the word "WOLF"}\}$

Firstly, convert the given sets in roster form and then check whether they have exactly the same elements.

Sol. (i) Here, $A = \{2, 3\}$

and $B = \{x : x \text{ is a solution of } x^2 + 5x + 6 = 0\}$

Firstly, we find the solution of $x^2+5x+6=0$.

By splitting the middle term, we get

$$x^2+3x+2x+6=0$$

$\Rightarrow \quad x(x+3)+2(x+3)=0$

$\Rightarrow \quad (x+2)(x+3)=0 \Rightarrow x=-2,-3$

$\therefore \quad B=\{-2,-3\}$

Since, the elements of sets A and B are not same, therefore $A \neq B$.

(ii) Here, $A=\{x: x$ is a letter of the word "FOLLOW"$\}$
$=\{\text{F, O, L, W}\}$

and $B=\{y: y$ is a letter of the word "WOLF"$\}$
$=\{\text{W, O, L, F}\}$

Since, every element of A is in B and every element of B is in A, i.e. both have exactly same elements.

$\therefore \quad A=B$

EXAMPLE |9| Which of the following pairs of sets are equal? Justify your answer.

(i) $A=\{x: x$ is a letter of the word "LOYAL"$\}$,
$B=\{x: x$ is a letter of the word "ALLOY"$\}$

(ii) $A=\{x: x \in Z$ and $x^2 \leq 8\}$,
$B=\{x: x \in R$ and $x^2-4x+3=0\}$

Sol. (i) Given, $A=\{x: x$ is a letter of the word "LOYAL"$\}$
$=\{\text{L, O, Y, A}\}$

and $B=\{x: x$ is a letter of the word "ALLOY"$\}$
$=\{\text{A, L, O, Y}\}$

Here, we see that both sets have exactly the same elements.

$\therefore \quad A=B$

(ii) Given, $A=\{x: x \in Z$ and $x^2 \leq 8\}=\{-2,-1,0,1,2\}$

[$\because$ the squares of integers $0, \pm 1, \pm 2$ are less than 8]

and $B=\{x: x \in R$ and $x^2-4x+3=0\}=\{1,3\}$

$[\because x^2-4x+3=0 \Rightarrow (x-1)(x-3)=0 \Rightarrow x=1,3]$

Here, we see that set A has 5 distinct elements and set B has 2 distinct elements. So, they do not have same elements.

$\therefore \quad A \neq B$

EXAMPLE |10| Identify which of the following set is an empty set, singleton set, infinite set or equal set.

(i) $A=\{x: x$ is a girl being living on the Jupiter$\}$

(ii) $B=\{x: x$ is a letter in the word "MARS"$\}$

(iii) $C=\{y: y$ is a letter in the word "ARMS"$\}$

(iv) $D=\{x: 3x-2=0, x \in Q\}$

(v) $E=\{x: x \in N$ and x is an odd number$\}$

Sol. (i) $A=\{x: x$ is a girl being living on the Jupiter$\}$

We know that, there is no human being or any girl living on the Jupiter. Therefore, A is an empty set.

(ii) $B=\{x: x$ is a letter in the word "MARS"$\}$

$\Rightarrow \quad B=\{\text{M, A, R, S}\}$

(iii) $C=\{y: y$ is a letter in the word "ARMS"$\}$

$\Rightarrow \quad C=\{\text{A, R, M, S}\}$

Here, we observe that the elements of sets B and C are exactly same, Therefore, these sets are equal, i.e. $B=C$.

(iv) $D=\{x: 3x-2=0, x \in Q\}$

$\Rightarrow \quad D=\left\{\frac{2}{3}\right\} \quad \left[\because 3x-2=0 \Rightarrow x=\frac{2}{3} \in Q\right]$

$\therefore D$ is a singleton set.

(v) $E=\{x: x \in N$ and x is an odd number$\}$

Clearly, it is an infinite set because there are infinite natural numbers which are odd.

TOPIC PRACTICE 2

OBJECTIVE TYPE QUESTIONS

1 The empty set is represented by

I. ϕ II. $\{\phi\}$

III. $\{\ \}$ IV. $\{\{\ \}\}$

(a) I and II   (b) I and III

(c) II and III   (d) I and IV

2 Identify the null set from the following

I. $\{x: x$ is a natural number, $x<5$ and $x>7\}$

II. $\{y: y$ is a point common to any two parallel lines$\}$

(a) Only I   (b) Only II

(c) Both I and II   (d) None of these

3 The set $\{x: x^2=x, x \in N\}$ can be expressed in roster form as

(a) $\{0,1\}$ (b) $\{1\}$ (c) $\{0\}$ (d) $\{\ \}$

4 Let S be any set and $n(S)$ is a natural number. Then, the set S is ...Y... . Here, Y refers to

(a) empty set   (b) infinite set

(c) non-empty finite set   (d) None of these

5 Let $A=\{x: x \in Z$ and $x^2 \leq 4\}$ and $B=\{x: x \in R$ and $x^2-3x+2=0\}$. Then,

(a) $A=B$ (b) $A \neq B$ (c) $A \in B$ (d) $A \notin B$

VERY SHORT ANSWER Type Questions

6 Which of the following sets are empty sets?

(i) $A=\{x: 4<x<5, x \in N\}$

(ii) $D=\{x: x^2=25$ and x is an odd integer$\}$

(iii) $\{x: x \in N$ and $x^2=9\}$

(iv) $\{x: x^2-3=0, x$ is rational$\}$

(v) $A=$ Set of odd natural numbers divisible by 2.

(vi) $B=$ Set of odd prime numbers.

7 Find the pairs of equal sets, from the following sets, if any, giving reasons.
$A = \{0\}$, $B = \{x : x > 15 \text{ and } x < 5\}$,
$C = \{x : x - 5 = 0\}$,
$D = \{x : x^2 = 25\}$,
$E = \{x : x$ is an integral positive root of the equation $x^2 - 2x - 15 = 0\}$ [NCERT]

8 Show that the set of letters needed to spell "CATARACT" and the set of letters needed to spell "TRACT" are equal.

9 Show that the following sets are equal.
$A = \{2, 1\}$, $B = \{2, 1, 1, 2, 1, 2\}$
and $C = \{x : x^2 - 3x + 2 = 0\}$

10 State which of the following sets are finite and which are infinite?
(i) $A = \{x : x \in Z \text{ and } x^2 - 2x - 3 = 0\}$
(ii) $B =$ Set of lines passing through a point.

SHORT ANSWER Type Questions

11 From the sets given below, select equal sets and equivalent sets.
$A = \{0, a\}$, $B = \{1, 2, 3, 4\}$, $C = \{4, 8, 12\}$,
$D = \{3, 1, 2, 4\}$, $E = \{1, 0\}$, $F = \{8, 4, 12\}$, $G = \{1, 5, 7, 11\}$,
$H = \{a, b\}$

12 Are the following sets equal?
$A = \{x : x$ is a letter in the word 'REAP'$\}$
$B = \{x : x$ is a letter in the word 'PAPER'$\}$
$C = \{x : x$ is a letter in the word 'ROPE'$\}$

13 Which of the following sets are equal?
$A = \{x : x \in N, x < 3\}$, $B = \{1, 2\}$, $C = \{3, 1\}$,
$D = \{x : x \in N, x < 5, x \text{ is odd}\}$, $E = \{1, 2, 1, 1\}$
$F = \{1, 1, 3\}$

14 State which of the following sets are finite and which are infinite?
(i) $A = \{x : x \in Z \text{ and } x^2 - 5x + 6 = 0\}$
(ii) $B = \{x : x \in Z \text{ and } x^2 \text{ is even}\}$
(iii) $C = \{x : x \in Z \text{ and } x^2 = 36\}$
(iv) $D = \{x : x \in Z \text{ and } x > -10\}$

15 From the sets given below, select empty set, singleton set, infinite set and equal sets.
(i) $A = \{x : x < 1 \text{ and } x > 3\}$
(ii) $B = \{x : x^3 - 1 = 0, x \in R\}$
(iii) $C = \{x : x \in N \text{ and } x \text{ is a prime number}\}$
(iv) $D = \{2, 4, 6, 8, 10\}$
(v) $E = \{x : x$ is a positive even integers and $x \le 10\}$

16 Which of the following sets are empty sets? [NCERT]
(i) $A = \{x : x$ is a human being living on the Mars$\}$
(ii) $B = \{x : x$ is an even natural number divisible by 3$\}$
(iii) $C = \{x : x$ is a point common to opposite sides of parallelogram$\}$
(iv) $D = \{x : x$ is a natural number, $x < 5$ and simultaneously $x > 7\}$

17 Which of the following sets are singleton/non-singleton?
(i) $A = \{x : |x| = 7, x \in N\}$
(ii) $B = \{x : x^2 + 2x + 1 = 0, x \in N\}$
(iii) $C = \{x : x^2 = 9, |x| \le 3, x \in N\}$

18 Which of the following pairs of sets are equal?
(i) $A = \{1, 3, 3, 1\}$, $B = \{1, 4\}$
(ii) $A = \{x : x + 2 = 2\}$, $B = \{0\}$
(iii) $A = \left\{1, \frac{1}{2}, \frac{1}{3}, \ldots\right\}$, $B = \left\{\frac{1}{n}, n \in N\right\}$
(iv) $A = \{x : x \in W\}$, $B = \{x : x \in N\}$

HINTS & ANSWERS

1. (b) The empty set is represented by ϕ or { }.

2. (c) There is no natural number which is simultaneously greater than 7 and less than 5.
Also, there is no point common to any two parallel lines.

3. (b) The natural number whose square is the number itself is 1. So, the given set in the roster form is {1}.

4. (c) If $n(S)$ is a natural number, then S is a non-empty finite set.

5. (b) $A = \{-2, -1, 0, 1, 2\}$
Given equation is $x^2 - 3x + 2 = 0$
$\Rightarrow \quad (x - 1)(x - 2) = 0 \Rightarrow x = 1, 2$
$\therefore \quad B = \{1, 2\}$
Since, $0 \in A$ and $0 \notin B$, A and B are not equal sets.

6. (i) There is no element in this set, so A is empty set.
(ii) $D = \{-5, 5\}$, not empty
(iii) $\{x : x \in N \text{ and } x^2 = 9\} = \{3\}$, so it is non-empty set.
(iv) Empty set
(v) As no odd natural number is divisible by 2, so set A is empty.
(vi) Since, 2 is only an even prime number.
$\therefore \quad B = \{2\}$
So, B is not an empty set.

7. $A = \{0\}$, $B = \{\ldots 3, 4 \text{ and } 16, 17, \ldots\}$
$C = \{5\}$, $D = \{5, -5\}$ and $E = \{5\}$
Since, each element of set C represents in set E and *vice-versa*.
Ans. Sets C and E are equal set.

8. Set of letters needed to spell "CATARACT" $= \{C, A, T, R\}$

Set of letters needed to spell "TRACT" $= \{T, R, A, C\}$

Both sets are same, so both are equal.

9. Given, $A = \{2, 1\}$, $B = \{2, 1, 1, 2, 1, 2\} = \{1, 2\}$

and $C = \{x : x^2 - 3x + 2 = 0\}$

$= \{x : (x-1)(x-2) = 0\} = \{1, 2\}$

From these, we observe that the sets A, B and C contain the same elements and hence these are equal.

10. (i) $A = \{x : x \in Z \text{ and } x^2 - 2x - 3 = 0\} = \{3, -1\}$

Here, A has two elements. So, it is a finite set.

(ii) Since, infinite number of lines can pass through a point. So C is an infinite set.

11 Equal sets : $B = D$, $C = F$

Equivalent sets : $A, E, H; B, D, G; C, F$ $\quad [\because n(A) = n(B)]$

12 No

13 $A = B = E$, $C = D = F$

14 (i) $A = \{2, 3\}$, so A is a finite set.

(ii) $B = \{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$, so B is on Infinite set.

(iii) $C = \{6, -6\}$, so C is a finite set.

(iv) $D = \{-9, -8, -7, \ldots\}$, so D is an infinite set.

15 (i) $A = \{\} = \phi$, so it is empty set.

(ii) $B = \{1\}$, so it is singleton set.

(iii) $C = \{2, 3, 5, \ldots\}$, so it is infinite set.

(iv) $D = \{2, 4, 6, 8, 10\}$

(v) $E = \{2, 4, 6, 8, 10\}$

D and E are equal set.

16. (i) $A = \{x : x \text{ is a human being living on Mars}\}$

We know that, there is no human being on the Mars. So, this set is an empty set.

(ii) $B = \{x : x \text{ is an even natural number divisible by } 3\}$

We know that, there is no even number, which is divisible by 3. So, this is an empty set.

(iii) $C = \{x : x \text{ is a point common to any two parallel lines}\}$

We know that, there is no common point between opposite sides of the parallelogram. So, this is an empty set.

(iv) $D = \{x : x \text{ is a natural number, } x < 5 \text{ and simultaneously } x > 7\}$

We know that, any natural number cannot be less than 5 and greater than 7, simultaneously.

So, this is an empty set.

17. (i) We have, $A = \{x : |x| = 7, x \in N\} = \{7\}$,

$[\because |x| = 7 \Rightarrow x = \pm 7, \text{ but } x \in N]$

This set is a singleton set.

(ii) We have, $B = \{x : x^2 + 2x + 1 = 0, x \in N\}$

Now, $\quad x^2 + 2x + 1 = 0$

$\Rightarrow \quad (x+1)^2 = 0 \Rightarrow x = -1$

which is not a natural number.

$\therefore \quad B = \{\} = \phi$

This set is non-singleton set

(iii) We have, $C = \{x : x^2 = 9, |x| \le 3, x \in N\}$

Now, $\quad x^2 = 9$

$\Rightarrow \quad x = \pm 3$

Now, $\quad |x| \le 3$

$\Rightarrow \quad x = -3, -2, -1, 0, 1, 2, 3$

Here, $x = -3$ and 3 satisfy $x^2 = 9$ and $|x| \le 3$ both.

But out of $-3, 3$ only $3 \in N$.

$\therefore \quad C = \{3\}$

This set is a singleton set.

18. (i) Given, $A = \{1, 3, 3, 1\} = \{1, 3\}$, $B = \{1, 4\}$

Since, A and B have different elements.

$\therefore \quad A \ne B$

(ii) Given, $A = \{x : x + 2 = 2\} = \{0\}$ and $B = \{0\}$

Since, A and B have same elements.

$\therefore \quad A = B$

(iii) Given, $A = \left\{1, \frac{1}{2}, \frac{1}{3}, \ldots\right\} = \left\{\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \ldots\right\}$

and $\quad B = \left\{\frac{1}{n} : n \in N\right\} = \left\{\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \ldots\right\}$

$= \left\{1, \frac{1}{2}, \frac{1}{3}, \ldots\right\}$

Since, A and B have same elements.

$\therefore \quad A = B$

(iv) Given, $A = \{x : x \in W\}$ and $B = \{x : x \in N\}$

$\Rightarrow A = \{0, 1, 2, 3, \ldots\}$ and $B = \{1, 2, 3, \ldots\}$

Since, A has one element differ from B i.e. 0.

$\therefore \quad A \ne B$

|TOPIC 3|

Subset and Universal Set

SUBSET

Let A and B be two sets. If every element of A is an element of B, then A is called a **subset** of B.

If A is a subset of B, then we write $A \subseteq B$, which is read as "A is a subset of B" or A is contained in B.

In other words, $A \subseteq B$, if whenever $a \in A$, then $a \in B$. It is often convenient to use the symbol "$\Rightarrow$" which means *implies*. Using this symbol, we can write the definition of subset as follows

$$A \subseteq B, \text{ if } x \in A \Rightarrow \quad x \in B$$

The above statement is read as

A is subset of B, if x is an element of A, then it implies that x is also an element of B.

If A is not a subset of B, then we write $A \not\subseteq B$.

e.g. Consider the sets A and B, where set A denotes the set of all students in your class, B denotes the set of all students in your school. We observe that, every element of A is also an element of B. Therefore, we can say that A is subset of B i.e. $A \subseteq B$.

If it happens for both sets A and B, i.e. every element of A is in B and every element of B is in A, then in this case, A and B are same sets. Thus we have,

$A \subseteq B$ and $B \subseteq A \Leftrightarrow A = B$, where "$\Leftrightarrow$" is a symbol for two ways implications and is usually read as if and only if (iff).

Note

(i) $\{1\} \subseteq \{1, 2, 3\}$
(ii) $2 \not\subseteq \{1, 2, 3\}$

Proper Subset

If $A \subseteq B$ and $A \neq B$, then A is called a proper subset of B, written as $A \subset B$ and B is called **superset** of A.

e.g. Let $A = \{x : x \text{ is an even natural number}\}$

and $\quad B = \{x : x \text{ is a natural number}\}$

Then, $\quad A = \{2, 4, 6, 8, \ldots\}$ and $B = \{1, 2, 3, 4, 5, \ldots\}$

$\therefore \quad A \subset B$

SOME IMPORTANT RESULTS

(i) Every set is a subset of itself.

Proof Let A be any set. Then, each element of A is clearly in A. Hence, $A \subseteq A$.

(ii) The empty set ϕ is a subset of every set.

Proof Let A be any set and ϕ be the empty set. To show that $\phi \subseteq A$, we have to show that every element of ϕ is an element of A also. But we know that empty set ϕ contains no element. So, there is no element of ϕ which does not belong to A. Thus, we can say that every element of ϕ is in A. Hence, $\phi \subseteq A$.

(iii) The total number of subsets and proper subset of a finite set containing n elements is 2^n and $2^n - 1$, respectively.

(iv) If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

(v) $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$.

EXAMPLE |1| Let $A = \{1, 2, \{3, 4\}, 5\}$. Which of the following statements are incorrect and why? **[NCERT]**

(i) $\{3, 4\} \subset A$ (ii) $\{3, 4\} \in A$
(iii) $\{\{3, 4\}\} \subset A$ (iv) $1 \in A$
(v) $1 \subset A$ (vi) $\{1, 2, 5\} \subset A$
(vii) $\{1, 2, 5\} \in A$ (viii) $\phi \subset A$
(ix) $\phi \in A$ (x) $\{\phi\} \subset A$

Sol. We have, $A = \{1, 2, \{3, 4\}, 5\}$

(i) Since, $\{3, 4\}$ is a member of set A.
$\therefore \quad \{3, 4\} \in A$
Hence, $\{3, 4\} \subset A$ is incorrect.

(ii) Since, $\{3, 4\}$ is a member of set A.
$\therefore \{3, 4\} \in A$ is correct.

(iii) Since, $\{3, 4\}$ is a member of set A.
So, $\{\{3, 4\}\}$ is a subset of A.
Hence, $\{\{3, 4\}\} \subset A$ is correct.

(iv) Since, 1 is a member of A.
$\therefore \quad 1 \in A$ is correct.

(v) Since, 1 is a member of set A.
$\therefore \quad 1 \subset A$ is incorrect.

(vi) Since, 1, 2, 5 are members of set A.
So, $\{1, 2, 5\}$ is a subset of set A.
Hence, $\{1, 2, 5\} \subset A$ is correct.

(vii) Since, 1, 2 and 5 are members of set A.
So, $\{1, 2, 5\}$ is a subset of A.
Hence, $\{1, 2, 5\} \in A$ is incorrect.

(viii) Since, ϕ is subset of every set.
$\therefore \quad \phi \subset A$ is correct.

(ix) Since, ϕ is not a member of set A.
$\therefore \quad \phi \in A$ is incorrect.

(x) Since, ϕ is not a member of set A.
$\therefore \{\phi\} \subset A$ is incorrect.

EXAMPLE |2| Insert the correct symbol $\subset$ or $\not\subset$ between each of the following pair of sets.

(i) $\{x : x$ is a student of class XI of your school$\}$ ...... $\{x : x$ is a student of your school$\}$ **[NCERT]**

(ii) $\{x : x$ is a triangle in the plane$\}$ ...... $\{x : x$ is a rectangle in the plane$\}$

(iii) $\{x : x$ is an equilateral triangle in the plane$\}$ ...... $\{x : x$ is a triangle in the plane$\}$ **[NCERT]**

(iv) $\{x : x$ is an even natural number$\}$ ...... $\{x : x$ is an integer$\}$

(v) $\{1, 4, 8\}$ ...... $\{1, 2, 4, 6, 8\}$

Sol. (i) $\{x : x$ is a student of class XI of your school$\}$ $\subset \{x : x$ is a student of your school$\}$

(ii) $\{x : x$ is a triangle in the plane$\} \not\subset \{x : x$ is a rectangle in the plane$\}$

[$\because$ all elements of Ist set are not there in the IInd set]

(iii) $\{x : x$ is an equilateral triangle in the plane$\} \subset \{x : x$ is a triangle in the plane$\}$

[$\because$ elements in the IInd set are triangles, so there will also be equilateral triangle, thus all elements of Ist set are there in the IInd set]

(iv) $\{x : x$ is an even natural number$\} \subset \{x : x$ is an integer$\}$

[$\because$ integers have both even and odd natural numbers, so all elements of Ist set will be there in the IInd set]

(v) $\{1, 4, 8\} \subset \{1, 2, 4, 6, 8\}$

EXAMPLE |3| In each of the following, determine whether the statement is true or false. If it is true, prove it and if it is false, give an example.

(i) If $x \in P$ and $P \in Q$, then $x \in Q$.

(ii) If $P \subset Q$ and $Q \subset R$, then $P \subset R$.

(iii) If $x \in P$ and $P \not\subset Q$, then $x \in Q$.

(iv) If $P \subset Q$ and $Q \in R$, then $P \in R$.

(v) If $P \not\subset Q$ and $Q \not\subset R$, then $P \not\subset R$.

Sol. (i) It is false statement.

e.g. Let $P = \{1\}$, $Q = \{\{1\}, 2\}$

Clearly, $1 \in P$ and $P \in Q$ but $1 \notin Q$.

So, $x \in P$ and $P \in Q$ need not imply that $x \in Q$.

(ii) It is true statement.

Let $x \in P$ be any arbitrary element, then

$P \subset Q \Rightarrow x \in Q$ and $Q \subset R \Rightarrow x \in R$

Thus, $x \in P \Rightarrow x \in R$ for all $x \in P \Rightarrow P \subset R$

Hence, $P \subset Q$ and $Q \subset R \Rightarrow P \subset R$

(iii) It is false statement.

e.g. Let $P = \{1, 2\}$ and $Q = \{2, 3, 4, 5\}$

Clearly, $1 \in P$ and $P \not\subset Q$ but $1 \notin Q$.

Thus, $x \in P$ and $P \not\subset Q$ need not imply that $x \in Q$.

(iv) It is false statement.

e.g. Let $P = \{1\}$, $Q = \{1, 2\}$ and $R = \{\{1, 2\}, 3\}$, then $P \subset Q$ and $Q \in R$ but $P \notin R$.

Thus, $P \subset Q$ and $Q \in R$ need not imply that $P \in R$.

(v) It is false statement.

e.g. Let $P = \{1, 2\}$, $Q = \{2, 3\}$ and $R = \{1, 2, 5\}$

Then, $P \not\subset Q$ and $Q \not\subset R$ but $P \subset R$.

Thus, $P \not\subset Q$ and $Q \not\subset R$ need not imply that $P \not\subset R$.

EXAMPLE |4| Prove that if $A \subseteq \phi$, then $A = \phi$.

Sol. Given, $A \subseteq \phi$ ...(i)

We know that ϕ is subset of every set.

$\therefore$ $\phi \subseteq A$ ...(ii)

From Eqs. (i) and (ii), we get

$A = \phi$ **Hence proved.**

EXAMPLE |5| Two finite sets have m and n elements. The number of subsets of the first set is 112 more than that of the second set. Find the values of m and n. **[NCERT Exemplar]**

Sol. Let the two sets be A and B such that $n(A) = m$ and $n(B) = n$.

Then, number of subsets of set $A = 2^m$

and number of subsets of set $B = 2^n$.

According to the given condition, we have

$2^m = 112 + 2^n \Rightarrow 2^m - 2^n = 2^7 - 2^4$

On comparing both sides, we get

$2^m = 2^7$ and $2^n = 2^4 \Rightarrow m = 7$ and $n = 4$

Subsets of the Set of Real Numbers

We know that every real number is either a **rational** or an **irrational** number and the set of real numbers is denoted by R. *There are many important subsets of set of real numbers which are given below*

NATURAL NUMBERS

The numbers being used in counting as 1, 2, 3, 4,..., called natural numbers.

The set of natural numbers is denoted by N. Thus,

$N = \{1, 2, 3, 4, \dots\}$

WHOLE NUMBERS

The natural numbers along with number 0 (zero) form the set of whole numbers i.e. 0, 1, 2, 3, ... , are whole numbers. The set of whole numbers is denoted by W. Thus,

$$W = \{0, 1, 2, 3, \dots\}$$

Note Set of natural numbers is the proper subset of the set of whole numbers.

i.e. $N \subset W$

INTEGERS

The natural numbers, their negatives and zero make the set of integers and it is denoted by Z.

$$Z = \{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$$

Note Set of whole numbers is the proper subset of integers.

i.e. $W \subset Z$

RATIONAL NUMBERS

A number of the form $\frac{p}{q}$, where p and q both are integers and $q \neq 0$ (division by 0 is not permissible), is called a rational number.

The set of rational numbers is generally denoted by Q.

Thus, $$Q = \left\{\frac{p}{q} : p, q \in Z \text{ and } q \neq 0\right\}$$

Note (i) All integers are also rational numbers, since any integer, say n, can be represented as the ratio $\frac{n}{1}$.

(ii) The set of integers is the proper subset of the set of rational numbers i.e. $Z \subset Q$ and hence $N \subset W \subset Z \subset Q$.

IRRATIONAL NUMBERS

A number which cannot be written in the form p/q, where p and q both are integers and $q \neq 0$, is called an irrational number.

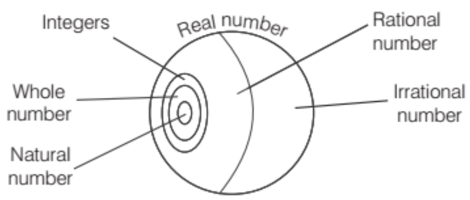
The set of irrational numbers is denoted by Q'. Thus,

$$Q' = \{x : x \in R \text{ and } x \notin Q\}$$

Note A non-terminating, non-repeating decimal number are irrational number.

DIAGRAMMATICAL REPRESENTATION

Above subsets can be represented diagrammatically as given below



INTERVALS AS SUBSETS OF *R*

Let a, b belongs to R and $a < b$. Then, the set of real numbers $\{x : a < x < b\}$ is called an **open interval** and is denoted by (a, b).

All the real numbers between a and b belongs to the open interval (a, b) but a and b do not belong to this set (interval).

The interval which contains the end point (a and b) also, is called **closed interval** and it is denoted by $[a, b]$.

Hence, $[a, b] = \{x : a \leq x \leq b\}$

Semi-closed or Semi-open Interval

Some intervals are closed at one end and open at the other, such intervals are called semi-closed or semi-open interval.

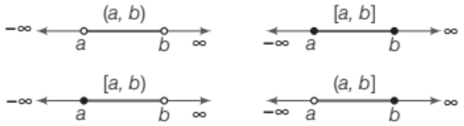
$[a, b) = \{x : a \leq x < b\}$ is a semi-open interval from a to b which includes a but excludes b.

$(a, b] = \{x : a < x \leq b\}$ is a semi-open interval from a to b which excludes a but includes b.

e.g. (i) $(2, 8)$ is a subset of $(-1, 11)$.

(ii) $[4, 6)$ is a subset of $[4, 6]$.

On real line, we can draw the interval, which is shown by the dark portion on the number line



Note

(i) The set $[0, \infty)$ defines the set of non-negative real numbers.

(ii) The set $(-\infty, 0)$ defines the set of negative real numbers.

(iii) $(-\infty, \infty)$ is the set of real numbers.

LENGTH OF AN INTERVAL

The number $(b - a)$ is called the length of any of the intervals (a, b), $[a, b]$, $[a, b)$ or $(a, b]$.

EXAMPLE |6| Write the following as intervals and also represent on the number line.

(i) $\{x : x \in R, -5 < x \leq 6\}$ (ii) $\{x : x \in R, -11 < x < -9\}$
(iii) $\{x : x \in R, 2 \leq x < 8\}$ (iv) $\{x : x \in R, 5 \leq x \leq 6\}$

Sol. (i) $\{x : x \in R, -5 < x \leq 6\}$ is the set that does not contain -5 but contains 6. So, it can be written as a semi-closed interval whose first end point is open and last end point is closed i.e. $(-5, 6]$.

On the real line, $(-5, 6]$ can be graphed as shown in figure given below

The dark portion on the number line represent $(-5, 6]$.

(ii) $\{x : x \in R, -11 < x < -9\}$ is the set that neither contains -11 nor -9, so it can be represented as open interval i.e. $(-11, -9)$.

On the real line, $(-11, -9)$ can be graphed as shown in figure given below

The dark portion on the number line represent $(-11, -9)$.

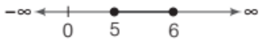
(iii) $\{x : x \in R, 2 \leq x < 8\}$ is the set that contains 2 but not contain 8. So, it can be represented as a semi-open interval whose first end point is closed and the other end point is open i.e. $[2, 8)$.

On the real line, $[2, 8)$ can be graphed as shown in figure given below

The dark portion on the number line represent $[2, \infty)$.

(iv) $\{x : x \in R, 5 \leq x \leq 6\}$ is the set which contains 5 and 6 both. So, it is equivalent to a closed interval i.e. $[5, 6]$.

On the real line, $[5, 6]$ can be graphed as shown in the figure given below



The dark portion on the number line represent $[5, 6]$.

EXAMPLE |7| Write the following intervals in set-builder form.

(i) $(-3, 0)$ (ii) $[6, 12]$ (iii) $(6, 12]$ (iv) $[-23, 5)$

Sol. We know that closed interval contains end points while open interval does not contain end points.

(i) $(-3, 0) = \{x : x \in R, -3 < x < 0\}$
(ii) $[6, 12] = \{x : x \in R, 6 \leq x \leq 12\}$
(iii) $(6, 12] = \{x : x \in R, 6 < x \leq 12\}$
(iv) $[-23, 5) = \{x : x \in R, -23 \leq x < 5\}$

UNIVERSAL SET

If there are some sets under consideration, then a set can be chosen arbitrarily which is a **superset** of each one of the given sets. Such a set is known as the universal set and it is denoted by U.

e.g. (i) Let $A = \{2, 4, 6\}$, $B = \{1, 3, 5\}$ and $C = \{0, 7\}$
Then, $U = \{0, 1, 2, 3, 4, 5, 6, 7\}$ is an universal set.

(ii) For the set of all integers, the universal set can be the set of rational numbers or the set of real numbers.

EXAMPLE |8| What universal sets would you propose for each of the following? **[NCERT]**

(i) The set of right triangles.
(ii) The set of equilateral triangles.

Sol. (i) The universal set for the set of right triangles is set of triangles.
(ii) The universal set for the set of equilateral triangle is set of isosceles triangles or set of triangles.

TOPIC PRACTICE 3

OBJECTIVE TYPE QUESTIONS

1 Consider
X = Set of all students in your school.
Y = Set of all students in your class.
Then, which of the following is true?
(a) Every element of Y is also an element of X
(b) Every element of X is also an element of Y
(c) Every element of Y is not an element of X
(d) Every element of X is not an element of Y

2 If $A \subset B$ and $A \neq B$, then
(a) A is called a proper subset of B
(b) A is called a super set of B
(c) A is not a subset of B
(d) B is a subset of A

3 Let A, B, C be three sets. If $A \in B$ and $B \subset C$, then
(a) $A \subset C$ (b) $A \in C$
(c) $A \notin C$ (d) None of these

4 The set of real numbers $\{x : a < x < b\}$ is called
(a) open interval
(b) closed interval
(c) semi-open interval
(d) semi-closed interval

5 The set of negative real numbers is denoted by ...Y... . Here, Y refers to
(a) $(-\infty, 0)$ (b) $[-\infty, 0]$
(c) $(-\infty, 0]$ (d) $[-\infty, 0)$

VERY SHORT ANSWER Type Questions

6 Consider the sets ϕ, $A = \{1, 2\}$ and $B = \{1, 4, 8\}$. Insert the following symbols $\subset$ or $\not\subset$ between each of the following pair of sets.
(i) $\phi \ldots B$ (ii) $A \ldots B$

7 Prove that $A \subseteq \phi$ implies $A = \phi$.

8 Write the following subset of R as interval. Also find the length of interval and represent on number line.
$\{x : x \in R, -12 \leq x \leq -10\}$

9 Let $A = \{a, b, \{c, d\}, e\}$. Which of the following statements is/are true?
(i) $\{c, d\} \in A$ (ii) $\{\{c, d\}\} \subset A$

10 Given, the sets $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ and $C = \{0, 2, 4, 6, 8\}$. Which of the following may be considered as universal set(s) for all three sets A, B and C? **[NCERT]**
(i) ϕ (ii) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

SHORT ANSWER Type Questions

11 Let $A = \{1, 3, 5\}$ and $B = \{x : x$ is an odd natural number $< 6\}$.
(i) Is $A \subseteq B$?
(ii) Is $A = B$?

12 Consider the following sets ϕ, $A = \{2, 5\}$, $B = \{1, 2, 3, 4\}$ and $C = \{1, 2, 3, 4, 5\}$
Insert the correct symbol $\subset$ or $\not\subset$ between each pair of sets
(i) $\phi \ldots B$ (ii) $A \ldots B$
(iii) $A \ldots C$ (iv) $B \ldots C$

13 If $A = \{3, \{4, 5\}, 6\}$, then find which of the following statements are true?

(i) $\{4, 5\} \subset A$ (ii) $\{4, 5\} \in A$
(iii) $\phi \subset A$ (iv) $\{3, 6\} \subset A$

14 Let A, B and C be three sets. If $A \in B$ and $B \subset C$, is it true that $A \subset C$? If not, give an example.

15 Write the following as intervals. **[NCERT]**

(i) $\{x : x \in R, -4 < x \le 6\}$
(ii) $\{x : x \in R, -12 < x < -10\}$
(iii) $\{x : x \in R, 0 \le x < 7\}$
(iv) $\{x : x \in R, 3 \le x \le 4\}$

16 Write the following intervals in set-builder form.

(i) $(-6, 0)$ (ii) $[3, 21)$
(iii) $[2, 21]$ (iv) $(-20, 5]$

17 Write down the subsets of the following sets.

(i) $\{1, 2, 3\}$
(ii) $\{\phi\}$ **[NCERT]**

18 If $A = \{x : x = n^2, n = 1, 2, 3\}$, then find the number of proper subsets.

19 Determine whether the given statements are true or false.

(i) If $A = \{3, 6, 7\}$ and $B = \{2, 3, 7, 8, 10\}$, then $A \subset B$.
(ii) If $A = \{x : x^2 + 4x - 21 = 0, x \in N\}$ and $B = \{-7, 3\}$, then $A \subseteq B$.
(iii) If $A = \{1, 7, 9\}$, $B = \{2, 4, 6\}$ and $C = \{0, 2, 4\}$, then $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is the universal set for all three sets.
(iv) $\{6\} \in \{4, 5, \{6\}, 7\}$

HINTS & ANSWERS

1. (a) We note that every element of Y is also an element of X, as if a student is in your class, then he is also in your school.

2. (a) If $A \subset B$ and $A \ne B$, then A is called a proper subset of B and B is called a super set of A.

3. (b) Let $A = \{1\}$, $B = \{\{1\}, 2\}$ and $C = \{\{1\}, 2, 3\}$.
Here, $A \in B$ and $A \in C$ but $A \not\subset C$ as $1 \in A$ but $1 \notin C$.

4. (a) Let $a, b \in R$ and $a < b$. Then, the set of real numbers $\{x : a < x < b\}$ is called an open interval, as a, b do not belong to this interval.

5. (a) The set of negative real numbers is denoted by $(-\infty, 0)$.

6. (i) Null set is a proper subset of every set.
Ans. $\phi \subset B$
(ii) $2 \in A$ but $2 \notin B$
Ans. $A \not\subset B$

7. Given, $A \subseteq \phi$ ...(i)
But $\phi \subseteq A$ [empty set is a subset of each set] ...(ii)
From Eqs. (i) and (ii), we get
$$A = \phi$$

8. $\{x : x \in R, -12 \le x \le -10\}$
Given set can be written in the interval from $[-12, -10]$

Ans. $[-12, -10]$, 2

9. (i) Since, a, b, $\{c, d\}$ and e are elements of A.
$\therefore$ $\{c, d\} \in A$
So, the statement is true.

20 Write the following as intervals and also represent on real line.

(i) $\{x : x \in R, -3 < x \le 7\}$
(ii) $\{x : x \in R, -11 < x < -7\}$
(iii) $\{x : x \in R, 0 \le x < 11\}$
(iv) $\{x : x \in R, 2 \le x \le 9\}$

21 Examine whether the following statements are true or false.

(i) $\{a, b\} \not\subset \{b, c, a\}$
(ii) $\{a, e\} \subset \{x : x$ is a vowel in the English alphabet$\}$
(iii) $\{a\} \in \{a, b, c\}$
(iv) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ is the universal set for the sets $\{1, 3, 5\}$ and $\{2, 4, 6\}$.

22 In each of the following, determine whether the statement is true or false. If it is true, then prove it. If it is false, then give an example.

(i) If $x \in A$ and $A \in B$, then $x \in B$.
(ii) If $A \subset B$ and $B \in C$, then $A \in C$.

(ii) As $\{c, d\} \in A$ and $\{\{c, d\}\}$ represents a set, which is a subset of A.
$\therefore$ $\{\{c, d\}\} \subset A$
So, the statement is true.

10. We know that, universal set for sets A, B and C is superset of A, B and C i.e. universal set contains all elements of A, B and C.
(i) ϕ cannot be considered as universal set.
(ii) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ is the universal set for the given sets.

11. (i) Given, $A = \{1, 3, 5\}$
and $B = \{x : x$ is an odd natural number $< 6\} = \{1, 3, 5\}$
So, $A \subseteq B$, because all elements of set A are present in set B.
Ans. Yes
(ii) Here, $B \subseteq A$, because all elements of set B are present in set A.
Hence, $A = B$, because both sets contain equal and same elements.
Ans. Yes

12. (i) ϕ is a proper subset of every set and $\phi \neq B$.

$\therefore \quad \phi \subset B$

(ii) $5 \in A$ but $5 \notin B$.

$\therefore \quad A \not\subset B$

(iii) Each element of A belongs to C and $A \neq C$.

$\therefore \quad A \subset C$

(iv) Each element of B also belongs to C and $B \neq C$

$\therefore \quad B \subset C$

13. (i) Since, element of any set is not a subset of any set and here $\{4, 5\}$ is an element of A.

So, statement is false.

(ii) $\{4, 5\} \in A$

So, statement is true.

(iii) $\phi \subset A$

So, statement is true.

(iv) $\{3, 6\}$ makes a set, so it is a subset of A i.e. $\{3, 6\} \subset A$.

So, statement is true.

14. Let $A = \{a\}$, $B = \{\{a\}, b\}$, $C = \{\{a\}, b, c\}$

Clearly, $A \in B$ and $B \subset C$. But $A \not\subset C$ as $a \in A$ but $a \notin C$

Thus, the given statement is not true.

15. (i) $\{x : x \in R, -4 < x \leq 6\}$ is the set that does not contain -4 but contain 6. So, it can be written as an interval whose first end is open and the last end is closed.

So, the interval is $(-4, 6]$.

(ii) $\{x : x \in R, -12 < x < -10\}$ is the set that neither contains -12 nor -10. So, it can be represented as an open interval. So, the interval is $(-12, -10)$.

(iii) $\{x : x \in R, 0 \leq x < 7\}$ is the set that contain 0 but not 7. So, it can be represented as an interval whose first end is closed and the other end is open. so, the interval is $[0, 7)$.

(iv) $\{x : x \in R, 3 \leq x \leq 4\}$ is the set which contain 3 and 4 both. So, it is equivalent to a closed interval.

(iv) $\{x : x \in R, -20 < x \leq 5\}$

17. (i) ϕ, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{2, 3\}$, $\{1, 3\}$, $\{1, 2, 3\}$

(ii) Clearly, $\{\phi\}$ is the power set of empty set ϕ. So, the subsets are ϕ and $\{\phi\}$.

18. Here, $\quad A = \{1, 4, 9\} \Rightarrow n(A) = 3$

Number of proper subsets $= 2^n - 1 = 7$

19. (i) False (ii) True (iii) True (iv) True

20. (i) $(-3, 7]$

(ii) $(-11, -7)$

(iii) $[0, 11)$

(iv) $[2, 9]$

21. (i) Since, the elements of the set $\{a, b\}$ are also present in the set $\{b, c, a\}$.

So, $\quad \{a, b\} \subset \{b, c, a\}$

So, statement is false.

(ii) Vowels in the English alphabets are a, e, i, o, u.

$\therefore \{a, e\} \subset \{x : x$ is a vowel in English alphabet$\}$

So, statement is true.

(iii) Since, $a \in \{a, b, c\}$ and not $\{a\}$.

So, statement is false.

(iv) Since, all elements of both sets $\{1, 3, 5\}$ and $\{2, 4, 6\}$ are subset of $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Hence, $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ is the universal set for given two sets.

So, the statement is true.

22. (i) Let $A = \{2\}$, $B = \{\{2\}, 3\}$

Clearly, $2 \in A$ and $A \in B$, but $2 \notin B$.

Thus, $x \in A$ and $A \in B$ need not imply that $x \in B$.

So, statements is false.

Ans. $[3, 4]$

16. (i) $\{x : x \in R, -6 < x < 0\}$

(ii) $\{x : x \in R, 3 \leq x < 21\}$

(iii) $\{x : x \in R, 2 \leq x \leq 21\}$

(ii) Let $A = \{2\}$, $B = \{2, 3\}$ and $C = \{\{2, 3\}, 4\}$

Clearly, $A \subset B$ and $B \in C$, but $A \notin C$.

Thus, $A \subset B$ and $B \in C$ need not imply that $A \in C$.

So, statement is false.

|TOPIC 4|

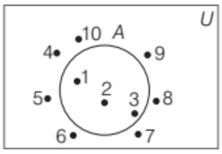
Venn Diagrams and Operations on Sets

VENN DIAGRAMS

Venn diagrams are named after the English logician, **John Venn (1834-1883)**. Venn diagrams represent most of the relationship between sets. These diagrams consist of rectangles and closed curves usually circles.

In Venn diagrams, the universal set is represented by a **rectangular region** and its subset are represented by **circle** or a **closed geometrical figure** inside the universal set. Also, an element of a set is represented by a **point** within the circle of set.

e.g. If $U = \{1, 2, 3, 4, \ldots, 10\}$ and $A = \{1, 2, 3\}$, *then its Venn diagram is as shown in the figure*



EXAMPLE |1| Draw the Venn diagrams to illustrate the following relationship among sets E, M and U, where E is the set of students studying English in a school, M is the set of students studying Mathematics in the same school and U is the set of all students in that school.

[NCERT Exemplar]

(i) All the students who study Mathematics also study English, but some students who study English do not

OPERATIONS ON SETS

There are some operations which when performed on two sets give rise to another set. Here, we will define certain operations on set and examine their properties.

Union of Sets

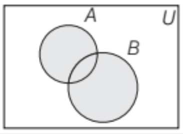
Let A and B be any two sets. The union of A and B is the set of all those elements which belong to either A or B or both. It is denoted by $A \cup B$ and read as **A union B**. The symbol '$\cup$' is used to denote the union.

$\therefore \quad A \cup B = \{x : x \in A \text{ or } x \in B\}$

e.g. Let $A = \{2, 3\}$ and $B = \{3, 4, 5\}$

Then, $A \cup B = \{2, 3, 4, 5\}$

The union of sets A and B is represented by the following Venn diagram



study Mathematics.

(ii) Not all students study Mathematics, but every student studying English studies Mathematics.

Sol. Given, E = Set of students studying English

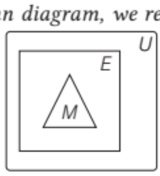
M = Set of students studying Mathematics

U = Set of all students

(i) Since, all of the students who study Mathematics also study English, but some students who study English do not study Mathematics.

$\therefore \quad M \subset E \subset U$

Through Venn diagram, we represent it as



(ii) Since, every student studying English studies Mathematics.

$\therefore \quad E \subset M \subset U$

Through Venn diagram, we represent it as

The shaded portion represents $A \cup B$.

EXAMPLE |2| Find the union of each of the following pairs of sets.

(i) $A = \{a, e, i, o, u\}, B = \{a, c, d\}$

(ii) $A = \{1, 3, 5\}, B = \{2, 4, 6\}$

(iii) $A = \{x : x \text{ is a natural number and } 1 < x \le 5\}$ and $B = \{x : x \text{ is a natural number and } 5 < x \le 10\}$

Sol. (i) Given, $A = \{a, e, i, o, u\}, B = \{a, c, d\}$

$\Rightarrow A \cup B = \{a, c, d, e, i, o, u\}$

(ii) Given, $A = \{1, 3, 5\}, B = \{2, 4, 6\}$

$\Rightarrow A \cup B = \{1, 2, 3, 4, 5, 6\}$

(iii) Given, $A = \{x : x \text{ is a natural number and } 1 < x \le 5\}$

$\Rightarrow \quad A = \{2, 3, 4, 5\}$

and $\quad B = \{x : x \text{ is a natural number and } 5 < x \le 10\}$

$\Rightarrow \quad B = \{6, 7, 8, 9, 10\}$

Now, $\quad A \cup B = \{2, 3, 4, 5\} \cup \{6, 7, 8, 9, 10\}$

$= \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$

EXAMPLE |3| Suppose $A_1, A_2, \ldots, A_{30}$ are thirty sets each having 5 elements and $B_1, B_2, \ldots, B_n$ are n sets each with 3 elements, let $\bigcup_{i=1}^{30} A_i = \bigcup_{i=1}^{n} B_j = S$ and each element of 'S' belongs to exactly 10 of the A_i's and exactly 9 of the B_j. Find 'n'. **[NCERT Exemplar]**

Sol. If elements are not repeated, then number of elements in $A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_{30}$ is 30×5.

But each element is used 10 times, so

$$n(S) = \frac{30 \times 5}{10} = 15$$

If elements in $B_1, B_2, \ldots, B_n$ are not repeated, then total number of elements is $3n$ but each element is repeated 9 times, so $n(S) = \frac{3n}{9}$

$\Rightarrow \quad 15 = \frac{3n}{9}$

$\Rightarrow \quad n = 45$

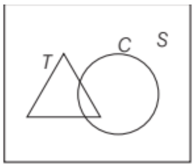
EXAMPLE |4| Let $S =$ Set of points inside the square, $T =$ Set of points inside the triangle and $C =$ Set of points inside the circle. If the triangle and circle intersect each other and are contained in a square. Then, prove that $S \cup T \cup C = S$, by Venn diagram. **[NCERT Exemplar]**

Sol. Given, $S =$ Set of points inside the square

$T =$ Set of points inside the triangle

and $C =$ Set of points inside the circle.

According to the given condition, the Venn diagram is given below



It is clear from the Venn diagram that, $S \cup T \cup C = S$

LAWS OF UNION OF SETS

For any three sets A, B and C, we have

(i) $A \cup \phi = A$ (Identity law)

(ii) $U \cup A = U$ (Universal law)

(iii) $A \cup A = A$ (Idempotent law)

(iv) $A \cup B = B \cup A$ (Commutative law)

(v) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)

EXAMPLE |5| If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$, then find

(i) $A \cup B \cup C$ (ii) $B \cup C \cup D$ **[NCERT]**

Sol. Given, $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$

(i) $A \cup B \cup C = (A \cup B) \cup C$

$= (\{1, 2, 3, 4\} \cup \{3, 4, 5, 6\}) \cup \{5, 6, 7, 8\}$

$= \{1, 2, 3, 4, 5, 6\} \cup \{5, 6, 7, 8\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

(ii) $B \cup C \cup D = (B \cup C) \cup D = (\{3, 4, 5, 6\} \cup \{5, 6, 7, 8\}) \cup \{7, 8, 9, 10\}$

$= \{3, 4, 5, 6, 7, 8\} \cup \{7, 8, 9, 10\}$

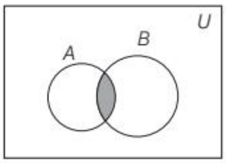
$= \{3, 4, 5, 6, 7, 8, 9, 10\}$

Intersection of Sets

Let A and B be any two sets. The intersection of A and B is the set of all those elements which belong to both A and B. It is denoted by $A \cap B$ and read as **A intersection B**. The symbol '$\cap$' is used to denote the intersection.

$\therefore \quad A \cap B = \{x : x \in A \text{ and } x \in B\}$

The intersection of sets A and B is represented by the following Venn diagram



The shaded portion represents $A \cap B$.

e.g. Let $A = \{2, 3, 4, 5\}$ and $B = \{1, 3, 6, 4\}$

Then, $A \cap B = \{3, 4\}$

EXAMPLE |6| Find the intersection of each of the following pairs of sets.

(i) $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 3, 6, 8, 9\}$

(ii) $A = \{e, f, g\}$, $B = \phi$

(iii) $A = \{x : x = 3n, n \in Z\}$, $B = \{x : x = 4n, n \in Z\}$,

Sol. (i) Given, $A = \{1, 3, 5, 7, 9\}$

and $B = \{2, 3, 6, 8, 9\}$

$\Rightarrow \quad A \cap B = \{3, 9\}$

[$\because$ 3 and 9 are only elements which are common]

(ii) Given, $A = \{e, f, g\}$ and $B = \phi$

$\Rightarrow \quad A \cap B = \phi$ [since, there is no common element]

(iii) Let $x \in A \cap B$

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow x$ is a multiple of 3 and x is a multiple of 4.

$\Rightarrow x$ is a multiple of 3 and 4 both.

$\Rightarrow x$ is a multiple of 12.

$\Rightarrow x = 12n, n \in Z$

$\therefore A \cap B = \{x : x = 12n, n \in Z\}$

EXAMPLE |7| If $a \in N$ such that $aN = \{an : n \in N\}$. Describe the set $3N \cap 7N$.

Sol. Given, $aN = \{an : n \in N\}$

$\therefore \quad 3N = \{3n : n \in N\}$
$= \{3, 6, 9, 12, 15, 18, 21, 24, 27, \ldots\}$

and $\quad 7N = \{7n : n \in N\}$
$= \{7, 14, 21, 28, 35, 42, \ldots\}$

Hence, $3N \cap 7N = \{21, 42, \ldots\} = \{21n : n \in N\} = 21N$

EXAMPLE |8| For any natural number a, we define $aN = \{an : n \in N\}$. If $b, c, d \in N$ such that $bN \cap cN = dN$, then prove that d is the LCM of b and c.

Sol. Given, $aN = \{an : n \in N\}$

= Set of positive integral multiple of a.

Similarly, $bN = \{bn : n \in N\}$

= Set of positive integral multiple of b

$cN = \{cn : n \in N\}$

= Set of positive integral multiple of c

$\therefore$ $bN \cap cN$ = Set of positive integral multiple of both b and c.

$\Rightarrow bN \cap cN = \{\lambda n : n \in N\}$, where λ is LCM of b and c.

Hence, d is the LCM of b and c. **Hence proved.**

LAWS OF INTERSECTION OF SETS

For any three sets, A, B and C, we have

(i) $A \cap \phi = \phi$ (Identity law)

(ii) $U \cap A = A$ (Universal law)

(iii) $A \cap A = A$ (Idempotent law)

(iv) $A \cap B = B \cap A$ (Commutative law)

Now, $B \cap C = \{4, 5, 9\} \cap \{1, 4, 6, 9\} = \{4, 9\}$

$\therefore \quad \text{RHS} = A \cap (B \cap C)$
$= \{4, 5, 7, 8, 10\} \cap \{4, 9\} = \{4\}$ ...(ii)

From Eqs. (i) and (ii), we get

LHS = RHS = $\{4\}$

Hence, $\quad (A \cap B) \cap C = A \cap (B \cap C)$

(ii) Here, $B \cap C = \{4, 9\}$

$\therefore \quad \text{LHS} = A \cup (B \cap C)$
$= \{4, 5, 7, 8, 10\} \cup \{4, 9\}$

$\Rightarrow \quad \text{LHS} = \{4, 5, 7, 8, 9, 10\}$ ...(iii)

Now, $A \cup B = \{4, 5, 7, 8, 10\} \cup \{4, 5, 9\}$
$= \{4, 5, 7, 8, 9, 10\}$

and $\quad A \cup C = \{4, 5, 7, 8, 10\} \cup \{1, 4, 6, 9\}$
$= \{1, 4, 5, 6, 7, 8, 9, 10\}$

$\therefore \quad \text{RHS} = (A \cup B) \cap (A \cup C)$
$= \{4, 5, 7, 8, 9, 10\} \cap \{1, 4, 5, 6, 7, 8, 9, 10\}$
$= \{4, 5, 7, 8, 9, 10\}$ ...(iv)

From Eqs. (iii) and (iv), we get

LHS = RHS = $\{4, 5, 7, 8, 9, 10\}$

Hence, $\quad A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(iii) Now, $B \cup C = \{4, 5, 9\} \cup \{1, 4, 6, 9\} = \{1, 4, 5, 6, 9\}$

$\therefore \quad \text{LHS} = A \cap (B \cup C)$
$= \{4, 5, 7, 8, 10\} \cap \{1, 4, 5, 6, 9\} = \{4, 5\}$ ...(v)

Now, $A \cap B = \{4, 5, 7, 8, 10\} \cap \{4, 5, 9\} = \{4, 5\}$

and $\quad A \cap C = \{4, 5, 7, 8, 10\} \cap \{1, 4, 6, 9\} = \{4\}$

$\therefore \quad \text{RHS} = (A \cap B) \cup (A \cap C) = \{4, 5\} \cup \{4\}$
$= \{4, 5\}$ ...(vi)

From Eqs. (v) and (vi), we get

LHS = RHS = $\{4, 5\}$

Hence, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(v) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law)

Distributive Law

If A, B and C are any three sets, then

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
i.e. intersection distributes over union.

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
i.e. union distributes over intersection.

EXAMPLE |9| If $A = \{4, 5, 7, 8, 10\}$, $B = \{4, 5, 9\}$ and $C = \{1, 4, 6, 9\}$, then verify that

(i) $(A \cap B) \cap C = A \cap (B \cap C)$

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(iii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Sol. Given, $A = \{4, 5, 7, 8, 10\}$,
$B = \{4, 5, 9\}$ and $C = \{1, 4, 6, 9\}$

(i) Now, $A \cap B = \{4, 5, 7, 8, 10\} \cap \{4, 5, 9\} = \{4, 5\}$

$\therefore \quad \text{LHS} = (A \cap B) \cap C$
$= \{4, 5\} \cap \{1, 4, 6, 9\} = \{4\}$ ...(i)

EXAMPLE |10| If $A = \{x : x \in N, x \le 7\}$, $B = \{x : x$ is prime, $x < 8\}$ and $C = \{x : x \in N$, x is odd and $x < 10\}$, verify that

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Sol. Given, $A = \{x : x \in N, x \le 7\} = \{1, 2, 3, 4, 5, 6, 7\}$
$B = \{x : x \text{ is prime}, x < 8\} = \{2, 3, 5, 7\}$

and $\quad C = \{x : x \in N, x \text{ is odd and } x < 10\} = \{1, 3, 5, 7, 9\}$

(i) Clearly, $A \cup B = \{1, 2, 3, 4, 5, 6, 7\} = A$,
$A \cup C = \{1, 2, 3, 4, 5, 6, 7, 9\}$,
$B \cap C = \{3, 5, 7\}$

Now, $A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6, 7\} = A$ ...(i)

and $(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6, 7\} = A$ ...(ii)

From Eqs. (i) and (ii), we get

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) Clearly, $A \cap B = \{2, 3, 5, 7\} = B$

$A \cap C = \{1, 3, 5, 7\}$, $B \cup C = \{1, 2, 3, 5, 7, 9\}$

Now, $\quad A \cap (B \cup C) = \{1, 2, 3, 5, 7\}$ ...(i) and $\quad (A \cap B) \cup (A \cap C) = \{1, 2, 3, 5, 7\}$ ...(ii)

From Eqs. (i) and (ii), we get

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

EXAMPLE |11| For any two sets A and B, prove that

(i) $A \cup (A \cap B) = A$ $\qquad$ (ii) $A \cap (A \cup B) = A$

Sol. (i) $A \cup (A \cap B) = (A \cup A) \cap (A \cup B)$ [by distributive law]

$= A \cap (A \cup B) = A$ $\quad [\because A \subseteq A \cup B]$

(ii) $A \cap (A \cup B) = (A \cap A) \cup (A \cap B)$ [by distributive law]

$= A \cup (A \cap B) = A$ $\quad [\because A \cap B \subseteq A]$

Hence proved.

EXAMPLE |12| Let A and B be two sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, then prove that $A = B$.

Sol. Given, $\quad A \cup X = B \cup X$

$\Rightarrow A \cap (A \cup X) = A \cap (B \cup X)$

$\Rightarrow \quad A = (A \cap B) \cup (A \cap X)$

$[\because A \subseteq A \cup X, A \cap (A \cup X) = A]$

$\Rightarrow \quad A = (A \cap B) \cup \phi \Rightarrow A = A \cap B$

$\Rightarrow \quad A \subseteq B$ $\quad [\because A \cap B \subseteq B]$ ...(i)

Again, $\quad A \cup X = B \cup X$

$\Rightarrow \quad B \cap (A \cup X) = B \cap (B \cup X)$

$\Rightarrow \quad (B \cap A) \cup (B \cap X) = B$

$[\because B \subseteq B \cup X, B \cap (B \cup X) = B]$

$\Rightarrow \quad (B \cap A) \cup \phi = B$ $\quad [\because B \cap X = \phi]$

$\Rightarrow \quad B \cap A = B$

$\Rightarrow \quad B \subseteq A$ $\quad [\because B \cap A \subseteq A]$ ...(ii)

From Eqs. (i) and (ii), we get

$A = B$ $\qquad$ **Hence proved.**

EXAMPLE |13| For any two sets A and B, prove that $A \cup B = A \cap B \Leftrightarrow A = B$.

Sol. Let $A = B$, then $A \cup B = A$ and $A \cap B = A$

$\Rightarrow \quad A \cup B = A \cap B$

Thus, $\quad A = B$

$\Rightarrow \quad A \cup B = A \cap B$ ...(i)

Conversely, let $A \cup B = A \cap B$

Now, let $\quad x \in A$

$\Rightarrow \quad x \in A \cup B$

$\Rightarrow \quad x \in A \cap B$ $\quad [\because A \cup B = A \cap B]$

$\Rightarrow \quad x \in A$ and $x \in B$

$\Rightarrow \quad x \in B$

$\therefore \quad A \subseteq B$ ...(ii)

Now, let $y \in B$

$\Rightarrow \quad y \in A \cup B$

$\Rightarrow \quad y \in A \cap B$ $\quad [\because A \cup B = A \cap B]$

$\Rightarrow \quad y \in A$ and $y \in B$

$\Rightarrow \quad y \in A$

$\therefore \quad B \subseteq A$ ...(iii)

From Eqs. (ii) and (iii), we get $A = B$

Thus, $\quad (A \cup B) = (A \cap B)$

$\Rightarrow \quad A = B$ ...(iv)

From Eqs. (i) and (iv), we get

$A \cup B = A \cap B \Leftrightarrow A = B$ $\qquad$ **Hence proved.**

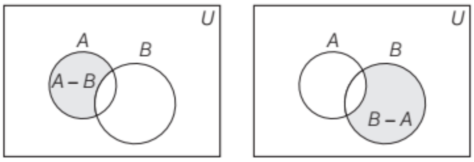
Difference of Sets

Let A and B be any two sets. The difference of sets A and B in this order is the set of all those elements of A which do not belong to B. It is denoted by $A - B$ and read as **A minus B**. The symbol '–' is used to denote the difference of sets.

$\therefore \quad A - B = \{x : x \in A \text{ and } x \notin B\}$

Similarly, $\quad B - A = \{x : x \in B \text{ and } x \notin A\}$

The difference of two sets A and B can be represented by the following Venn diagram



The shaded portion represents the difference of two sets A and B.

e.g. Let $\quad A = \{1, 2, 3, 4, 5\}$ and $B = \{3, 5, 7, 9\}$

Then, $\quad A - B = \{1, 2, 4\}$ and $B - A = \{7, 9\}$

EXAMPLE |14|

(i) If $X = \{a, b, c, d, e, f\}$ and $Y = \{f, b, d, g, h, k\}$, then find $X - Y$ and $Y - X$.

(ii) If $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6\}$, then find $A - B$ and $B - A$.

Also, represent each of these by Venn Diagram.

Sol. (i) Given, $\quad X = \{a, b, c, d, e, f\}$

and $\quad Y = \{f, b, d, g, h, k\}$

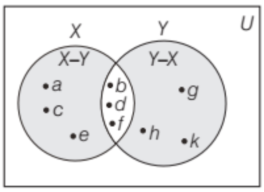
$\therefore X - Y = \{a, b, c, d, e, f\} - \{f, b, d, g, h, k\} = \{a, c, e\}$

[only those elements of X which do not belong to Y]

and $Y - X = \{f, b, d, g, h, k\} - \{a, b, c, d, e, f\}$

$= \{g, h, k\}$

[only those elements of Y which do not belong to X]



(ii) Given, $\quad A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6\}$

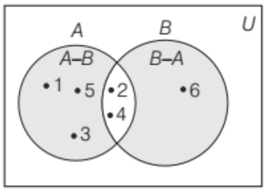
$\therefore \quad A - B = \{1, 2, 3, 4, 5\} - \{2, 4, 6\}$

$\Rightarrow \quad A - B = \{1, 3, 5\}$

[only those elements of A which do not belong to B]

and $\quad B - A = \{2, 4, 6\} - \{1, 2, 3, 4, 5\} = \{6\}$

[only those elements of B which do not belongs to A]



EXAMPLE |15| If A, B and C are three sets such that $A \subset B$, then prove that $C - B \subset C - A$.

Sol. Let $\quad x \in C - B \Rightarrow x \in C$ and $x \notin B$

$\Rightarrow \quad x \in C$ and $x \notin A \quad [\because A \subset B]$

$\Rightarrow \quad x \in C - A$

$\therefore \quad C - B \subset C - A$ **Hence proved.**

Symmetric Difference of Two Sets

Let A and B be any two sets. The symmetric difference of A and B is the set $(A - B) \cup (B - A)$. It is denoted by $A \Delta B$ and read as A **symmetric difference** B. The symbol 'Δ' is used to denote the symmetric difference.

$\therefore \quad A \Delta B = (A - B) \cup (B - A)$

$= \{x : x \in A \text{ or } x \in B \text{ but } x \notin A \cap B\}$

The symmetric difference of sets A and B is represented by the following Venn diagram

EXAMPLE |17| Let $A = \{1, 2, 4, 5\}$, $B = \{2, 3, 5, 6\}$, $C = \{4, 5, 6, 7\}$, verify that $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$.

Sol. Given, $A = \{1, 2, 4, 5\}$, $B = \{2, 3, 5, 6\}$ and $C = \{4, 5, 6, 7\}$

Now, $\quad A \cap B = \{2, 5\}$ and $A \cap C = \{4, 5\}$

and $\quad B \Delta C = (B - C) \cup (C - B) = \{2, 3, 4, 7\}$

$\therefore \quad A \cap (B \Delta C) = \{2, 4\}$ ...(i)

and $\quad (A \cap B) \Delta (A \cap C) = \{2, 4\}$ ...(ii)

From Eqs. (i) and (ii), we get

$A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$ **Hence proved.**

LAW OF DIFFERENCE OF SETS

(a) For any two sets A and B, we have

(i) $A - B = A \cap B'$

(ii) $B - A = B \cap A'$

(iii) $A - B \subseteq A$

(iv) $B - A \subseteq B$

(v) $A - B = A \Leftrightarrow A \cap B = \phi$

(vi) $(A - B) \cup B = A \cup B$

(vii) $(A - B) \cap B = \phi$

(viii) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

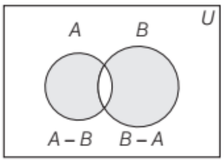
(b) If A, B and C are any three sets, then

(i) $A - (B \cap C) = (A - B) \cup (A - C)$

(ii) $A - (B \cup C) = (A - B) \cap (A - C)$

(iii) $A \cap (B - C) = (A \cap B) - (A \cap C)$

(iv) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$



Here, the shaded portion represents the symmetric difference of sets A and B.

e.g. Let $\quad A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$

Then, $\quad A - B = \{1, 2\}$, $B - A = \{5, 6\}$

Now, $\quad A \Delta B = (A - B) \cup (B - A) = \{1, 2, 5, 6\}$

EXAMPLE |16| Find the symmetric difference of sets $A = \{1, 3, 5, 6, 7\}$ and $B = \{3, 7, 8, 9\}$.

Sol. Given sets are $A = \{1, 3, 5, 6, 7\}$ and $B = \{3, 7, 8, 9\}$.

Now, $A - B = \{1, 3, 5, 6, 7\} - \{3, 7, 8, 9\} = \{1, 5, 6\}$

[set of those elements of A, which are not present in B]

and, $\quad B - A = \{3, 7, 8, 9\} - \{1, 3, 5, 6, 7\} = \{8, 9\}$

[set of those elements of B, which are not present in A]

$\therefore$ Required symmetric difference,

$A \Delta B = (A - B) \cup (B - A) = \{1, 5, 6\} \cup \{8, 9\} = \{1, 5, 6, 8, 9\}$

EXAMPLE |18| Prove that

$$A - (B \cap C) = (A - B) \cup (A - C).$$

Sol. To prove, $A - (B \cap C) = (A - B) \cup (A - C)$

Let $\quad x \in A - (B \cap C)$

$\Rightarrow \quad x \in A$ and $x \notin (B \cap C)$

$\Rightarrow \quad x \in A$ and $(x \notin B$ or $x \notin C)$

$\Rightarrow \quad (x \in A$ and $x \notin B)$ or $(x \in A$ and $x \notin C)$

$\Rightarrow \quad x \in (A - B)$ or $x \in (A - C)$

$\Rightarrow \quad x \in (A - B) \cup (A - C)$

$\therefore \quad A - (B \cap C) \subseteq (A - B) \cup (A - C)$ ...(i)

Again, let $y \in (A - B) \cup (A - C)$

$\Rightarrow \quad y \in (A - B)$ or $y \in (A - C)$

$\Rightarrow \quad (y \in A$ and $y \notin B)$ or $(y \in A$ and $y \notin C)$

$\Rightarrow \quad y \in A$ and $(y \notin B$ or $y \notin C)$

$\Rightarrow \quad y \in A$ and $y \notin (B \cap C) \Rightarrow y \in A - (B \cap C)$

$\therefore \quad (A - B) \cup (A - C) \subseteq A - (B \cap C)$ ...(ii)

From Eqs. (i) and (ii), we get

$A - (B \cap C) = (A - B) \cup (A - C)$ **Hence proved.**

EXAMPLE |19| Show that

(i) $A \cap (B - C) = (A \cap B) - (A \cap C)$

(ii) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

Sol. (i) Let x be any arbirary element of $A \cap (B - C)$.

Then, $x \in A \cap (B - C)$

$\Rightarrow$ $x \in A$ and $x \in B - C$

$\Rightarrow$ $x \in A$ and ($x \in B$ and $x \notin C$)

$\Rightarrow$ ($x \in A$ and $x \in B$) and ($x \in A$ and $x \notin C$)

$\Rightarrow$ $x \in (A \cap B)$ and $x \notin (A \cap C)$

$\Rightarrow$ $x \in (A \cap B) - (A \cap C)$

$\Rightarrow$ $A \cap (B - C) \subseteq (A \cap B) - (A \cap C)$ ...(i)

Now, let y be any arbitrary element of $(A \cap B) - (A \cap C)$

Then, $y \in (A \cap B) - (A \cap C)$

$\Rightarrow$ $y \in (A \cap B)$ and $y \notin (A \cap C)$

$\Rightarrow$ ($y \in A$ and $y \in B$) and ($y \in A$ and $y \notin C$)

$\Rightarrow$ $y \in A$ and ($y \in B$ and $y \notin C$)

$\Rightarrow$ $y \in A$ and $y \in B - C$

$\Rightarrow$ $y \in A \cap (B - C)$

$\Rightarrow$ $(A \cap B) - (A \cap C) \subseteq A \cap (B - C)$ ...(ii)

From Eqs. (i) and (ii), we get

$A \cap (B - C) = (A \cap B) - (A \cap C)$

(ii) LHS $= A \cap (B \Delta C) = A \cap [(B - C) \cup (C - B)]$

$[\because A \Delta B = (A - B) \cup (B - A)]$

$= [A \cap (B - C)] \cup [A \cap (C - B)]$

$[\because \cap$ distribute over $\cup]$

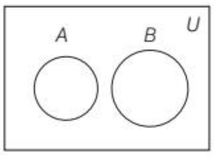
$= [(A \cap B) - (A \cap C)] \cup [(A \cap C) - (A \cap B)]$

$= (A \cap B) \Delta (A \cap C) =$ RHS **Hence proved.**

Disjoint Sets

Two sets A and B are said to be disjoint sets, if they have **no common element** i.e. $A \cap B = \phi$.

The disjoint of two sets A and B can be represented by the Venn diagram



e.g. Let $A = \{2, 4, 6\}$ and $B = \{1, 3, 5\}$

Then, $A \cap B = \phi$. Hence, A and B are disjoint sets.

Note The sets $A - B$, $A \cap B$ and $B - A$ are mutually disjoint sets i.e. the intersection of any of these two sets is an empty set.

EXAMPLE |20| Which of the following pairs of sets are disjoint?

(i) $A = \{1, 2, 3, 4, 5, 6\}$
and $B = \{x : x$ is a natural number and $4 \le x \le 6\}$

(ii) $A = \{x : x$ is the boys of your school$\}$
$B = \{x : x$ is the girls of your school$\}$

Sol. (i) Given, $A = \{1, 2, 3, 4, 5, 6\}$
and $B = \{4, 5, 6\}$

$\therefore$ $A \cap B = \{1, 2, 3, 4, 5, 6\} \cap \{4, 5, 6\} = \{4, 5, 6\} \ne \phi$

Hence, this pair of set is not disjoint.

(ii) Here, $A = \{b_1, b_2, \dots b_n\}$ and $B = \{g_1, g_2, \dots g_m\}$
where, $b_1, b_2, \dots, b_n$ are the boys and $g_1, g_2, \dots, g_m$ are the girls of school.

Clearly, $A \cap B = \phi$

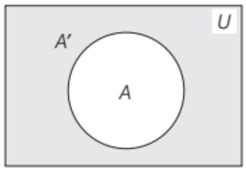
Hence, this pair of set is disjoint set.

Complement of a Set

Let U be the universal set and A be any subset of U, then complement of A with respect to U is the set of all those elements of U which are not in A. It is denoted by $\overline{A}$ or A' and read as, '**A complement**'.

Thus, $\overline{A} = \{x : x \in U$ and $x \notin A\}$ or $A' = U - A$

The complement of set A is represented by the following Venn diagram



Here, the shaded portion represents the complement of set A.

e.g. Let $U = \{1, 2, 3, 4, 5, 6\}$

and $A = \{2, 4\}$

Then, $A' = U - A = \{1, 2, 3, 4, 5, 6\} - \{2, 4\}$
$= \{1, 3, 5, 6\}$

Note If A is a subset of the universal set U, then its complement A' is also a subset of U.

Some Properties of Complement of Sets

(i) $(A')' = A = U - A'$ [law of double complementation]

(ii) (a) $A \cup A' = U$

(b) $A \cap A' = \phi$ [complement laws]

(iii) (a) $\phi' = U$

(b) $U' = \phi$ [laws of empty set and universal set]

(iv) $(A \cup B)' = U - (A \cup B)$

EXAMPLE |21| Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$. Find

(i) A' (ii) B' (iii) $(A \cap C)'$

(iv) $(A \cup B)'$ (v) $(A')'$ (vi) $(B - C)'$

Sol. Given, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$

$B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$

(i) $A' = U - A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 3, 4\}$
$= \{5, 6, 7, 8, 9\}$

(ii) $B' = U - B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 4, 6, 8\}$
$= \{1, 3, 5, 7, 9\}$

(iii) $A \cap C = \{3, 4\}$

$\therefore\ (A \cap C)' = U - (A \cap C)$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{3, 4\} = \{1, 2, 5, 6, 7, 8, 9\}$

(iv) $A \cup B = \{1, 2, 3, 4, 6, 8\}$

$(A \cup B)' = U - (A \cup B) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$- \{1, 2, 3, 4, 6, 8\} = \{5, 7, 9\}$

(v) $(A')' = U - A' = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{5, 6, 7, 8, 9\}$

$= \{1, 2, 3, 4\}$ [using part (i)]

Alternatively

We know that, $(A')' = A$

$\therefore\ (A')' = \{1, 2, 3, 4\}$

(vi) Now, $B - C = \{2, 8\}$

$(B - C)' = U - (B - C)$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 8\}$

$= \{1, 3, 4, 5, 6, 7, 9\}$

Theorem (De Morgan's Law)

Statement If A and B are any two sets, then

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Proof (i) Let x_1 be any arbitrary element of $(A \cup B)'$.

Then, $x_1 \in (A \cup B)'$

$\Rightarrow\ x_1 \notin (A \cup B)$

$\Rightarrow\ x_1 \notin A$ and $x \notin B$

$\Rightarrow\ x_1 \in A'$ and $x \in B'$

$\Rightarrow\ x_1 \in A' \cap B'$

$\therefore\ (A \cup B)' \subseteq A' \cap B'$ [$\because x_1$ is arbitrary]...(i)

Again, let $x_2 \in A' \cap B'$

$\Rightarrow\ x_2 \in A'$ and $x_2 \in B'$

$\Rightarrow\ x_2 \notin A$ and $x_2 \notin B$

$\Rightarrow\ x_2 \notin A \cup B$

$\Rightarrow\ x_2 \in (A \cup B)'$

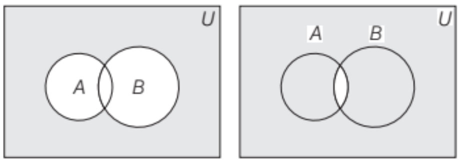
$\therefore\ A' \cap B' \subseteq (A \cup B)'$ [$\because x_2$ is arbitrary] ...(ii)

From Eqs. (i) and (ii), we get

$(A \cup B)' = A' \cap B'$

Similarly, we can prove part (ii).

The Venn diagram of De Morgan's law is shown below



(i) (ii)

EXAMPLE |22| Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$.

Verify that **[NCERT]**

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Use the formulae, $(A \cup B)' = U - (A \cup B)$, $A' = U - A$ and $B' = U - B$ and then simplify it.

Sol. (i) We have, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$

$\therefore\ A \cup B = \{2, 4, 6, 8\} \cup \{2, 3, 5, 7\}$

$= \{2, 3, 4, 5, 6, 7, 8\}$

Now, $(A \cup B)' = U - (A \cup B)$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$- \{2, 3, 4, 5, 6, 7, 8\}$

$= \{1, 9\}$ ...(i)

Now, $A' = U - A$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 4, 6, 8\}$

$= \{1, 3, 5, 7, 9\}$

and $B' = U - B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 3, 5, 7\}$

$= \{1, 4, 6, 8, 9\}$

$\therefore\ A' \cap B' = \{1, 3, 5, 7, 9\} \cap \{1, 4, 6, 8, 9\} = \{1, 9\}$ ...(ii)

From Eqs. (i) and (ii), we get

$(A \cup B)' = A' \cap B'$

(ii) Here, $A \cap B = \{2, 4, 6, 8\} \cap \{2, 3, 5, 7\} = \{2\}$

$\therefore\ (A \cap B)' = U - (A \cap B)$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2\}$

$\Rightarrow\ (A \cap B)' = \{1, 3, 4, 5, 6, 7, 8, 9\}$ ...(i)

Now, $A' \cup B' = \{1, 3, 5, 7, 9\} \cup \{1, 4, 6, 8, 9\}$

$= \{1, 3, 4, 5, 6, 7, 8, 9\}$ ...(ii)

From Eqs. (i) and (ii), we get

$(A \cap B)' = A' \cup B'$

EXAMPLE |23| Prove that $(A \cap B')' \cup (B \cap C) = A' \cup B$. **[NCERT Exemplar]**

Sol. LHS $= (A \cap B')' \cup (B \cap C)$

$= \{A' \cup (B')'\} \cup (B \cap C)$ [by De Morgan's law]

$= (A' \cup B) \cup (B \cap C)$ [$\because (B')' = B$]

$= ((A' \cup B) \cup B) \cap ((A' \cup B) \cup C)$

$= (A' \cup (B \cup B)) \cap (A' \cup B \cup C)$

$= (A' \cup B) \cap (A' \cup B \cup C)$

$= (A' \cup B) =$ RHS **Hence proved.**

EXAMPLE |24| For any two sets A and B, prove that $A \cap B = \phi$ implies $A \subseteq B'$.

Sol. Let $A \cap B = \phi$

Now, let $x \in A$

$\Rightarrow\ x \notin B$ [$\because A \cap B = \phi$]

$\Rightarrow\ x \in B'$

$\Rightarrow\ A \subseteq B'$ [$\because x$ is arbitrary]

Thus, $A \cap b = \phi$

$\Rightarrow\ A \subseteq B'$ **Hence proved.**

EXAMPLE |25| Using properties of sets, show that for any two sets A and B, $(A \cup B) \cap (A \cup B') = A$.

Sol. LHS $= (A \cup B) \cap (A \cup B')$

$= A \cup (B \cap B')$ [by distributive law]

$= A \cup \phi$ [$\because B \cap B' = \phi$]

$= A =$ RHS [by identity law]

Hence proved.

EXAMPLE |26| Let A and B be two sets. Prove that $(A - B) \cup B = A$ if and only if $B \subset A$.

Sol. Let $(A - B) \cup B = A$

$\Rightarrow (A \cap B') \cup B = A$ [$\because A - B = A \cap B'$]

$\Rightarrow (A \cup B) \cap (B' \cup B) = A$ [by distributive law]

$\Rightarrow (A \cup B) \cap U = A$ [$\because B' \cup B = U$]

$\Rightarrow A \cup B = A \Rightarrow B \subset A$

Conversely, let $B \subset A$

$\therefore (A - B) \cup B = (A \cap B') \cup B$

$= (A \cup B) \cap (B' \cup B)$

[by distributive law]

$= (A \cup B) \cap U$ [$\because B' \cup B = U$]

$= A \cup B = A$ [$\because B \subset A$] **Hence proved.**

EXAMPLE |27| Let A, B and C be three sets such that $A \cup B = C$ and $A \cap B = \phi$. Then, prove that $A = C - B$.

Sol. We have, $A \cup B = C$

$\therefore C - B = (A \cup B) - B = (A \cup B) \cap B'$ [$\because X - Y = X \cap Y'$]

$= (A \cap B') \cup (B \cap B')$ [by distributive law]

$= (A \cap B') \cup \phi$

$= A \cap B' = A - B$

$= A$ [$\because A \cap B = \phi$] **Hence proved.**

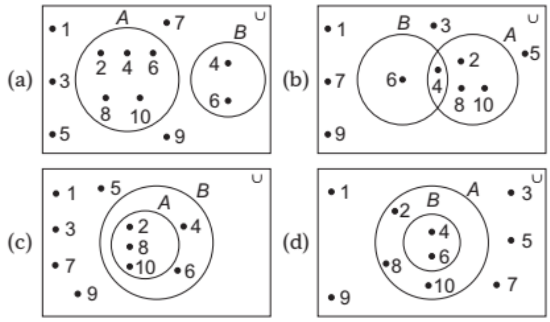
TOPIC PRACTICE 4

OBJECTIVE TYPE QUESTIONS

1 Most of the relationships between sets can be represented by means of diagrams which are known as

(a) rectangles
(b) circles
(c) Venn diagrams
(d) triangles

2 If $U = \{1, 2, 3, 4, \ldots, 10\}$ is the universal set of A, B where $A = \{2, 4, 6, 8, 10\}$ and $B = \{4, 6\}$. Then given sets can be represented by Venn diagram as



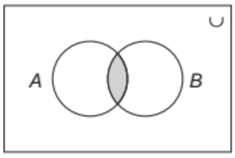
3 Which of the following represent the union of two sets A and B?

(a) $\{x : x \in A\}$ (b) $\{x : x \in A \text{ and } x \in B\}$
(c) $\{x : x \in B\}$ (d) $\{x : x \in A \text{ or } x \in B\}$

4 Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $B = \{2, 3, 5, 7\}$. Then, which of following is true?

(a) $A \cap B = A$ (b) $A \cap B = B$
(c) $A \cap B \not\subset B$ (d) None of these

5 The shaded portion represented in the Venn diagram is



(a) $A \cup B$ (b) $A \cap B$ (c) $A - B$ (d) $B - A$

VERY SHORT ANSWER Type Questions

6 Let $A = \{2, 4, 6, 8\}$ and $B = \{6, 8, 10, 12\}$. Find $A \cup B$.

7 If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$ and $C = \{11, 13, 15\}$, then find

(i) $A \cap B$ (ii) $B \cap C$

8 If $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 3, 5, 7, 11\}$, then find $A - B$.

9 Find the smallest set A such that

$$A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}.$$

10 Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 3\}$, $B = \{2, 4, 6, 7\}$ and $C = \{2, 3, 4, 8\}$, then find $(B \cup C)'$, $(A \cup B)'$. **[NCERT Exemplar]**

11 If $A = \{x : x \text{ is a positive multiple of } 3\}$ and $U = \{x : x \text{ is a natural number}\}$, then find A'.

12 Represent the following sets in Venn diagram.

(i) $A' \cap (B \cup C)$ (ii) $A' \cap (C - B)$

13 If $B' \subseteq A'$, then show that $A \subseteq B$.

SHORT ANSWER Type I Questions

14 Find the union of each of the following pair of sets.

(i) $A = \{a, e, i, o, u\}$, $B = \{a, b, c\}$

(ii) $A = \{1, 3, 5\}$, $B = \{1, 2, 3\}$

15 Find $A \cap B$, if $A = \{3, 5, 7, 9, 11\}$ and $B = \{7, 9, 11, 13\}$. Also, represent it by Venn diagram. **[NCERT]**

16 If A and B are two sets such that $A \subset B$, then show that $A \cup B = B$. **[NCERT]**

17 Let $A = \{x : x$ is a natural number$\}$ and $B = \{x : x$ is an even natural number$\}$. Find $A \cap B$.

18 Let $A = \{x : x \in N\}$, $B = \{x : x = 2n, n \in N\}$, $C = \{x : x = 2n - 1, n \in N\}$ and $D = \{x : x$ is a prime natural number$\}$. Find

(i) $A \cap B$ (ii) $A \cap C$

(iii) $B \cap C$ (iv) $B \cap D$

19 For three sets A, B and C, show that $A \cap B = A \cap C$ need not imply $B = C$.

20 For any two sets A and B, show that the following statements are equivalent.

(i) $A \subset B$ (ii) $A - B = \phi$

(iii) $A \cup B = B$ (iv) $A \cap B = A$

21 State whether each of the following statement is true or false.

(i) $A = \{2, 4, 6, 8\}$ and $B = \{1, 3, 5\}$ are disjoint sets.

(ii) $A = \{a, e, i, o, u\}$ and $B = \{a, b, c, d\}$ are disjoint sets.

26 If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 3, 4\}$ and $B = \{5, 6\}$, verify that $A - B = A \cap B' = B' - A'$.

27 If $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 6\}$ and $C = \{5, 6, 7, 8\}$, then verify that $(A \cap B)' = A' \cup B'$.

28 If $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17\}$, $B = \{2, 4, \dots, 18\}$ and N, the set of natural numbers is the universal set, then prove that $A' \cup \{(A \cup B) \cap B'\} = N$.

29 Taking the set of a natural numbers as the universal set, write down the complements of the following sets. **[NCERT]**

(i) $\{x : x$ is a natural number divisible by 3 and 5$\}$

(ii) $\{x : x$ is a perfect square$\}$

(iii) $\{x : x$ is a perfect cube$\}$

(iv) $\{x : x + 5 = 8\}$ (v) $\{x : 2x + 5 = 9\}$

(vi) $\{x : x \geq 7\}$ (vii) $\{x : x \in N, 2x + 1 > 10\}$

30 If $X = \{a, b, c, d\}$ and $Y = \{f, b, d, g\}$, then find **[NCERT]**

(i) $X \cup Y$ (ii) $X \cap Y$

(iii) $X - Y$ (iv) $Y - X$

Also, represent each of these by Venn diagram.

31 If $U = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$, $A = \{2, 4, 7\}$, $B = \{3, 5, 7, 9, 11\}$ and $C = \{7, 8, 9, 10, 11\}$, then compute

(i) $(A \cap U) \cap (B \cup C)$ (ii) $C - B$

(iii) $B - C$

SHORT ANSWER Type II Questions

22 Let $A = \{3, 6, 12, 15, 18, 21\}$, $B = \{4, 8, 12, 16, 20\}$, $C = \{2, 4, 6, 8, 10, 12, 14, 16\}$ and $D = \{5, 10, 15, 20\}$. Find

(i) $A - B$ (ii) $A - C$

(iii) $B - C$ (iv) $B - D$

23 Let $A = \{1, 2, 4, 5\}$, $B = \{2, 3, 5, 6\}$, $C = \{4, 5, 6, 7\}$. Verify that

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

24 If $A = \{a, b, c, d, e\}$, $B = \{a, c, e, g\}$ and $C = \{b, e, f, g\}$, then verify that

(i) $A \cap (B - C) = (A \cap B) - (A \cap C)$

(ii) $A - (B \cap C) = (A - B) \cup (A - C)$

25 If A and B are any two sets, then prove that $(A \cap B) \cup (A - B) = A$.

32 Find $A \Delta B$, if

(i) $A = \{1, 3, 4\}$ and $B = \{2, 5, 9, 11\}$.

(ii) $A = \{1, 3, 6, 11, 12\}$ and $B = \{1, 6\}$.

33 Which of the following pairs of sets are disjoint **[NCERT]**

(i) $\{1, 2, 3, 4\}$ and $\{x : x$ is a natural number and $4 \leq x \leq 6\}$

(ii) $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$

(iii) $\{x : x$ is an even integer$\}$ and $\{x : x$ is an odd integer$\}$

34 Let $A = \{x : x \in N$ and x is a multiple of 2$\}$

$B = \{x : x \in N$ and x is a multiple of 5$\}$

and $C = \{x : x \in N$ and x is multiple of 10$\}$

Describe the sets

(i) $(A \cap B) \cap C$ (ii) $A \cup (B \cap C)$ (iii) $A \cap (B \cup C)$

35 Give examples of three sets A, B and C such that $A \cap B \neq \phi$, $B \cap C \neq \phi$, $A \cap C \neq \phi$ and $A \cap B \cap C = \phi$.

36 If $A = \{a, b, c, d, e\}$, $B = \{a, c, e, g\}$ and $C = \{b, e, f, g\}$, verify that

(i) $B \cap C = C \cap B$ (ii) $A \cap C = C \cap A$

(iii) $(A \cup B) \cup C = A \cup (B \cup C)$

(iv) $A \cap (B \cap C) = (A \cap B) \cap C$

37 Let $A = \{1, 2, 4, 5\}$, $B = \{2, 3, 5, 6\}$, $C = \{4, 5, 6, 7\}$. Verify the following identities.

(i) $A \cap (B - C) = (A \cap B) - (A \cap C)$

(ii) $A - (B \cup C) = (A - B) \cap (A - C)$

(iii) $A - (B \cap C) = (A - B) \cup (A - C)$

38 Let A and B be two sets. Using properties of set, prove that

(i) $A \cap B = \phi \Rightarrow A \subseteq B$

(ii) $A' \cup B = U \Rightarrow A \subseteq B$

39 Let A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that $B = C$. **[NCERT]**

HINTS & ANSWERS

1. (c)

2. (d) Given $U = \{1, 2, 3, 4, \ldots, 10\}$

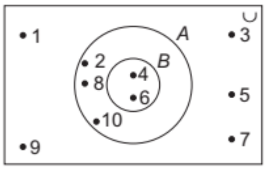
$A = \{2, 4, 6, 8, 10\}$

and $B = \{4, 6\}$

$\because$ All the elements of B are also in A.

$\therefore$ $B \subset A$

$\Rightarrow$ Set B lies inside A in the Venn diagram



3. (d)

4. (b) Given $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

and $B = \{2, 3, 5, 7\}$

$\therefore$ $A \cap B = \{2, 3, 5, 7\}$

$A \cap B = B$

5. (b) The intersection of two sets A and B is set of all those elements which belong to both A and B *i.e.*, $A \cap B$.

The shaded portion in the given figure indicates the intersection of A and B *i.e.*, $A \cap B$.

6. If $A = \{2, 4, 6, 8\}$, $B = \{6, 8, 10, 12\}$, then $A \cup B = \{2, 4, 6, 8, 10, 12\}$

7. Solve as Example 6 (i).

Ans. (i) $A \cap B = \{7, 9, 11\}$ (ii) $B \cap C = \{11, 13\}$

8. Solve as Example 14 (ii).

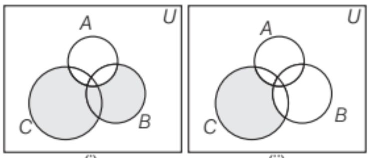
Ans. $A - B = \{1, 9\}$

9. $A = \{3, 5, 9\}$

10. $(B \cup C)' = \{1, 5, 9, 10\}$ and $(A \cup B)' = \{5, 8, 9, 10\}$

11. $\{x : x \in N$ and x is not a multiple of $3\}$

12.



(i) (ii)

13. Let $x \in A \Rightarrow x \notin A'$

$\Rightarrow$ $x \notin B'$ $[\because B' \subseteq A']$

$\Rightarrow$ $x \in B$

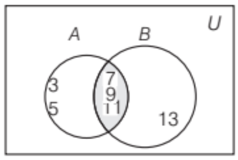
14. (i) $A \cup B = \{a, b, c, e, i, o, u\}$

(ii) $A \cup B = \{1, 2, 3, 5\}$

15. We have, $A = \{3, 5, 7, 9, 11\}$ and $B = \{7, 9, 11, 13\}$

$\because$ $A \cap B = \{3, 5, 7, 9, 11\} \cap \{7, 9, 11, 13\}$

$\therefore$ $A \cap B = \{7, 9, 11\}$



16. $A \subset B$ means that, all the elements of set A are in set B and also there are some other elements in B. We know that, $A \cup B$ contains all the elements either in A or in B or in both A and B. Thus, $A \cup B = B$

17. Given, $A = \{x : x$ is a natural number$\} = \{1, 2, 3, 4, \ldots\}$

and $B = \{x : x$ is an even natural number$\} = \{2, 4, 6, \ldots\}$

We observe that 2, 4, 6, ... are the elements which are common to both the sets A and B.

Ans. $A \cap B = \{2, 4, 6, \ldots\} = B$

18. (i) $A \cap B = \{x : x = 2n, n \in N\} = B$

(ii) $A \cap C = \{x : x = 2n - 1, n \in N\} = C$

(iii) $B \cap C = \{\,\} = \phi$

(iv) $B \cap D = \{2\}$

19. Let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$ and $C = \{1, 3, 4, 7, 8\}$

20. Let $A \subset B$, $A - B = \{x \in A$ but $x \notin B\}$

$\because A \subset B$, therefore there is no element in A which does not belong to B.

$\therefore$ $A - B = \phi$

Again, we have $A - B = \phi \Rightarrow A \subset B \Rightarrow A \cup B = B$

We have $A \cup B = B \Rightarrow A \subset B \Rightarrow A \cap B = A$

$A \cap B = A \Rightarrow A \subset B$

21. (i) We have, $A = \{2, 4, 6, 8\}$ and $B = \{1, 3, 5\}$

Now, $A \cap B = \{2, 4, 6, 8\} \cap \{1, 3, 5\}$

$\Rightarrow$ $A \cap B = \phi$

Therefore, A and B are disjoint sets.
So, the statement is true.

(ii) We have, $A = \{a, e, i, o, u\}$ and $B = \{a, b, c, d\}$
Now, $A \cap B = \{a\}$
i.e. $A \cap B \neq \phi$
Therefore, A and B are not disjoint sets.
So, the statement is false.

22. (i) $\{3, 6, 15, 18, 21\}$ (ii) $\{3, 15, 18, 21\}$
(iii) $\{20\}$ (iv) $\{4, 8, 12, 16\}$

23. Solve as Example 9.

24. Find LHS and RHS, separately, to verify the result.

25. LHS $= (A \cap B) \cup (A \cap B')$ $\quad [\because A - B = A \cap B']$

Let $A \cap B = X$, then

$$\text{LHS} = X \cup (A \cap B') = (X \cup A) \cap (X \cup B') \quad \ldots\text{(i)}$$
$[\because \cup$ distribute over $\cap]$

$\because \quad X = A \cap B$

$\therefore \quad X \cup A = (A \cap B) \cup A = A \quad [\because A \cap B \subseteq A]$

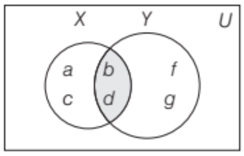
and $\quad X \cup B' = (A \cap B) \cup B'$
$= (A \cup B') \cap (B \cup B')$ $[\because \cup$ distribute over $\cap]$
$= (A \cup B') \cap U$ $\quad [\because B \cup B' = U]$
$= A \cup B'$

Now, substitute the value of $X \cup A$, $X \cup B'$ in Eq. (i), to get result

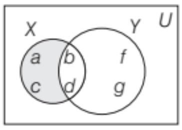
26. $A' = \{2, 5, 6, 7, 8, 9, 10\}$
$B' = \{1, 2, 3, 4, 7, 8, 9, 10\}$
$A \cap B' = \{1, 3, 4\}$ and $B' - A' = \{1, 3, 4\}$

27. $A \cap B = \{3, 4\} \Rightarrow (A \cap B)' = \{1, 2, 5, 6, 7, 8\}$
$A' = \{5, 6, 7, 8\}$ and $B' = \{1, 2, 5, 7, 8\}$
$A' \cup B' = \{1, 2, 5, 6, 7, 8\}$
Clearly, $(A \cap B)' = A' \cup B'$

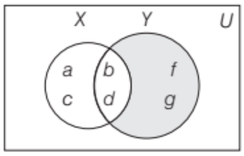
(ii) $X \cap Y = \{b, d\}$



(iii) $X - Y = \{a, c\}$



(iv) $Y - X = \{f, g\}$



31. (i) $(A \cap U) = \{2, 4, 7\} \cap \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$
$= \{2, 4, 7\}$
$B \cup C = \{3, 5, 7, 9, 11\} \cup \{7, 8, 9, 10, 11\}$
$= \{3, 5, 7, 8, 9, 10, 11\}$
$\therefore (A \cap U) \cap (B \cup C) = \{2, 4, 7\} \cap \{3, 5, 7, 8, 9, 10, 11\}$
$= \{7\}$

(ii) $C - B = \{7, 8, 9, 10, 11\} - \{3, 5, 7, 9, 11\}$
$= \{8, 10\}$

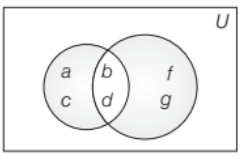
(iii) $B - C = \{3, 5, 7, 9, 11\} - \{7, 8, 9, 10, 11\} = \{3, 5\}$

32. (i) We have, $A = \{1, 3, 4\}$
and $B = \{2, 5, 9, 11\}$

28. $A \cup B = \{1, 2, 3, 4, 5, \ldots, 16, 17, 18\}$
$A' = \{2, 4, 6, 8, 10, \ldots, 18, 19, 20\ 21, 22, \ldots\}$
$B' = \{1, 3, 5, 7, 9, \ldots, 17, 19, 20, 21, 22, \ldots\}$
$\therefore (A \cup B) \cap B' = \{1, 3, 5, 7, 9, \ldots 17\}$

29. (i) $\{x : x \in N$ and x is not divisible by $15\}$
(ii) $\{x : x \in N$ and x is not a perfect square$\}$
(iii) $\{x : x \in N$ and x is not a perfect cube$\}$
(iv) $\{x : x \in N$ and $x \neq 3\}$
(v) $\{x : x \in N$ and $x \neq 2\}$
(vi) $\{x : x \in N$ and $x < 7\}$
(vii) $\left\{x : x \in N \text{ and } x \leq \frac{9}{2}\right\}$

30. (i) $X \cup Y = \{a, b, c, d, f, g\}$



Now, $(A - B) = \{1, 3, 4\} - \{2, 5, 9, 11\} = \{1, 3, 4\}$
and $(B - A) = \{2, 5, 9, 11\} - \{1, 3, 4\} = \{2, 5, 9, 11\}$
$\therefore \quad A \Delta B = (A - B) \cup (B - A)$
$= \{1, 3, 4\} \cup \{2, 5, 9, 11\}$
$= \{1, 2, 3, 4, 5, 9, 11\}$

(ii) Solve as part (i).
Ans. $A \Delta B = \{3, 11, 12\}$

33. (i) $\{x : x$ is a natural number and $4 \leq x \leq 6\} = \{4, 5, 6\}$
Now, $\{1, 2, 3, 4\}$ and $\{4, 5, 6\}$ have one element common.
So, it is not disjoint set.

(ii) The sets $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$ have one element 'e' common.
So, it is not disjoint set.

(iii) $E = \{x : x$ is an even integer$\} = \{\ldots, -4, -2, 0, 2, 4, 6, \ldots\}$
and $O = \{x : x$ is an odd integer$\}$
$= \{\ldots, -5, -3, -1, 1, 3, 5, \ldots\}$
So, it is disjoint set.

34. (i) Here, $A \cap B = \{2, 4, 6, \ldots\} \cap \{5, 10, 15, \ldots\}$
$= \{10, 20, 30 \ldots\} = C$

$\therefore (A \cap B) \cap C = C \cap C = C$

$= \{10, 20, 30, \ldots\}$

(ii) Here, $B \cap C = \{5, 10, 15, \ldots\} \cap \{10, 20, 30 \ldots\}$

$= \{10, 20, 30, \ldots\}$

$\therefore A \cup (B \cap C) = \{2, 4, 6, \ldots\} \cup \{10, 20, 30, \ldots\} = A$

$= \{2, 4, 6, 8, 10, \ldots\}$

(iii) Here, $B \cup C = \{5, 10, 15, \ldots\} \cup \{10, 20, 30, \ldots\}$

$= \{5, 10, 15, 20, 25, \ldots\}$

$\therefore A \cap (B \cup C) = \{2, 4, 6, \ldots\} \cap \{5, 10, 15, \ldots\} = C$

$= \{10, 20, 30, \ldots\}$

35. Let $A = \{1, 2\}$, $B = \{2, 3, 4\}$ and $C = \{1, 3, 5\}$

Then, $A \cap B = \{1, 2\} \cap \{2, 3, 4\} = \{2\} \neq \phi$

$B \cap C = \{2, 3, 4\} \cap \{1, 3, 5\} = \{3\} \neq \phi$

$A \cap C = \{1, 2\} \cap \{1, 3, 5\} = \{1\} \neq \phi$

and $A \cap B \cap C = \{1, 2\} \cap \{2, 3, 4\} \cap \{1, 3, 5\} = \phi$

36. Find LHS and RHS, separately, to verify the result.

37. Find LHS and RHS, separately, to verify the result.

38. (i) We have, $A = (A \cap U) = A \cap (B \cup B')$

$= (A \cap B) \cup (A \cap B') \cup]$

$= (A \cap B) \cup \phi$

$\Rightarrow A = A \cap B \Rightarrow A \subseteq B$

(ii) We have, $A' \cup B = U$

$\Rightarrow (A' \cup B)' = U'$

$\Rightarrow (A')' \cap B' = \phi$

$\Rightarrow A \cap B' = \phi$

Now, from part (i), we get

$A \subseteq B$

39. Given, $A \cup B = A \cup C$

$\Rightarrow (A \cup B) \cap C = (A \cup C) \cap C$

$\Rightarrow (A \cap C) \cup (B \cap C) = C$

$\Rightarrow (A \cap B) \cup (B \cap C) = C$

Again, $A \cup B = A \cup C$

$\Rightarrow (A \cup B) \cap B = (A \cup C) \cap B$

$\Rightarrow B = (A \cap B) \cup (C \cap B)$

$\Rightarrow B = (A \cap B) \cup (B \cap C)$

From Eqs. (i) and (ii), we get

$B = C$

|TOPIC 5|

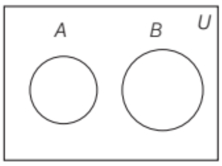
Applications of Set Theory

In this topic, we will discuss some word problems related to our daily life, which are based on union and intersection of two sets. Before solving these types of problem, we should know the following formulae.

If A and B are two finite sets, then two cases arise

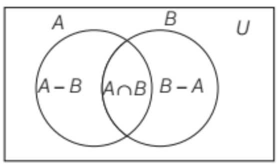
***Case* I** If A and B are **disjoint sets**, i.e. there is no common element in A and B i.e. $A \cap B = \phi$. Then,

$$n(A \cup B) = n(A) + n(B)$$



***Case* II** If A and B are **not disjoint sets**, then there are common elements in A and B. Then,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$



Note Sets $A - B$, $A \cap B$ and $B - A$ are disjoint and their union is $A \cup B$.

IMPORTANT RESULTS

(i) $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

(ii) $n(A) = n(A - B) + n(A \cap B)$

(iii) $n(B) = n(B - A) + n(A \cap B)$

(iv) $n(A \Delta B) = n[(A - B) \cup (B - A)]$

$= n(A - B) + n(B - A)$

[since, $(A - B)$ and $(B - A)$ are disjoint sets]

$= n(A) + n(B) - 2n(A \cap B)$

(v) $n(A' \cup B') = n[(A \cap B)'] = n(U) - n(A \cap B)$

(vi) $n(A' \cap B') = n[(A \cup B)'] = n(U) - n(A \cup B)$

(vii) $n(A - B) = n(A \cap B)' = n(A) - n(A \cap B)$

(viii) If A, B and C are finite sets, then

(a) $n(A \cup B \cup C)$

$= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(b) $n(A \text{ only}) = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

(c) $n(\overline{A} \cap \overline{B} \cap \overline{C}) = n(U) - n(A \cup B \cup C)$

Knowledge Plus

Let A, B and C be any three sets, then the meaning of different operations on these sets are given below.

(i) $A' \rightarrow$ Set of those objects which does not belong to A (similar meaning for B' and C').

(ii) $A \cup B \rightarrow$ Set of those objects which lies either in A or in B (similar meaning for $B \cup C$ and $A \cup C$).

(iii) $A \cap B \rightarrow$ Set of those objects which belongs to both the sets A and B (similar meaning for $B \cap C$ and $C \cap A$).

(iv) $A \cap B \cap C \rightarrow$ Set of those objects which belongs to A, B and C.

(v) $A \cup B \cup C \rightarrow$ Set of those objects which belongs to atleast one of the sets A, B and C.

EXAMPLE |1| If X and Y are two sets such that X has 40 elements, $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements, then how many elements does Y have?

Use the formula, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ and simplify it.

SOL. Given, $n(X) = 40$, $n(X \cup Y) = 60$

and $n(X \cap Y) = 10$

Clearly, $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$60 = 40 + n(Y) - 10$

$\Rightarrow \quad 60 = 30 + n(Y)$

$\Rightarrow \quad n(Y) = 60 - 30$

$\therefore \quad n(Y) = 30$

Hence, Y have 30 elements.

EXAMPLE |2| If X and Y are two sets such that $n(X) = 17$, $n(Y) = 23$ and $n(X \cup Y) = 38$, then find $n(X \cap Y)$.

SOL. Given, $n(X) = 17$, $n(Y) = 23$

and $n(X \cup Y) = 38$

Clearly, $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$\Rightarrow \quad n(X \cap Y) = n(X) + n(Y) - n(X \cup Y)$

$\Rightarrow \quad n(X \cap Y) = 17 + 23 - 38 = 40 - 38 = 2$

EXAMPLE |3| If $n(A) = 4$, $n(B) = 5$, $n(U) = 7$ and $n(A \cap B) = 2$, then find the value of $n(A \cup B)'$.

SOL. Given, $n(A) = 4$, $n(B) = 5$, $n(U) = 7$

and $n(A \cap B) = 2$

$\because \quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\therefore \quad n(A \cup B) = 4 + 5 - 2 = 7$

Now, $n(A \cup B)' = n(U) - n(A \cup B) = 7 - 7 = 0$

EXAMPLE |4| In a class of 60 students, 25 students play Cricket, 20 students play Tennis and 10 students play both the games. Then, find the number of students who play neither games. **[NCERT Exemplar]**

SOL. Let C and T respectively denote the set of students who play Cricket and Tennis, and set U denotes the set of all students in a class.

Then, $n(C) = 25$, $n(T) = 20$, $n(C \cap T) = 10$ and $n(U) = 60$

We know that, $n(C \cup T) = n(C) + n(T) - n(C \cap T)$

$\Rightarrow \quad n(C \cup T) = 25 + 20 - 10 = 45 - 10 = 35$

Now, the number of students who play neither game

$n(C' \cap T') = n(C \cup T)'$ [by De Morgan's law]

$= n(U) - n(C \cup T) = 60 - 35 = 25$

Hence, 25 students play neither games.

EXAMPLE |5| In a town of 840 persons, 450 persons read Hindi, 300 read English and 200 read both newspapers. Then, find the number of persons who read neither of the newspapers. **[NCERT Exemplar]**

SOL. Let H and E respectively denote the set of persons who read Hindi and English newspapers and let U set of all perseon in a town.

Then, we have $n(U) = 840$,

$n(H) = 450$,

$n(E) = 300$

and $n(H \cap E) = 200$

Clearly, $n(H \cup E) = n(H) + n(E) - n(H \cap E)$

$\therefore \quad n(H \cup E) = 450 + 300 - 200$

$= 750 - 200 = 550$

Now, the number of persons who read neither of the newspaper is given by

$n(H' \cap E') = n(H \cup E)'$ [by De Morgan's law]

$= n(U) - n(H \cup E)$

$= 840 - 550 = 290$

Hence, 290 persons read neither of the newspapers.

EXAMPLE |6| Each student in a class of 40 students study atleast one of the subjects English, Mathematics and Economics. 16 students study English, 22 Economics and 26 Mathematics, 5 study English and Economics, 14 Mathematics and Economics and 2 English, Economics and Mathematics. Find the number of students who study English and Mathematics.

Sol. Let A, B and C denote the set of students who study English, Economics and Mathematics, respectively.

Then, we have,

Total number of students, $n(A \cup B \cup C) = 40$

Number of students who study English, $n(A) = 16$

Number of students who study Economics, $n(B) = 22$

Number of students who study Mathematics, $n(C) = 26$

Number of students who study English and Economics, $n(A \cap B) = 5$

Number of students who study Mathematics and Economics, $n(B \cap C) = 14$

and number of students who study all subjects,

$n(A \cap B \cap C) = 2$

Clearly, $n(A \cup B \cup C) = n(A) + n(B) + n(C)$
$$- n(A \cap B) - n(B \cap C)$$
$$- n(A \cap C) + n(A \cap B \cap C)$$
$$40 = 16 + 22 + 26 - 5 - 14 - n(C \cap A) + 2$$
$\Rightarrow \quad 40 = 66 - 19 - n(C \cap A)$

$\Rightarrow \quad n(C \cap A) = 47 - 40 = 7$

Hence, number of students who study English and Mathematics are 7.

EXAMPLE |7| A market research group conducted a survey of 2000 consumers and reported that 1720 consumers liked "Patanjali Ghee" and 1450 consumers liked "Amul Ghee". What is the least number that must have liked both the product?

SOL. Let U be the set of consumers who were participated in the survey.

Then, $\quad n(U) = 2000$

Let P be the set of consumers who liked "Patanjali Ghee".

Then, $\quad n(P) = 1720$

Let A be the set of consumers who liked "Amul Ghee".

Then, $\quad n(A) = 1450$

$\because \quad n(P \cup A) = n(P) + n(A) - n(P \cap A)$

$\therefore \; n(P \cup A) = 1720 + 1450 - n(P \cap A) = 3170 - n(P \cap A)$

Now, as $\quad n(P \cup A) \le n(U)$

$\therefore \quad 3170 - n(P \cap A) \le 2000$

$\Rightarrow \quad n(P \cap A) \ge 1170$

Hence, the least number of consumers who liked both the product is 1170.

EXAMPLE |8| In a survey of 500 customer in an electronic shop, 400 were purchases LG refrigerator and 200 purchases SAMSUNG refrigerator, 50 purchase both refrigerators. Is this data correct?

SOL. Let U be the set of all customers who were participated in the survey. Then, $n(U) = 500$

Let denotes the set of customers who purchased LG refrigerator.

Then, $\quad n(L) = 400$

Let S denotes the set of customers who purchased SAMSUNG refrigerator.

Then $\quad n(S) = 200$

and $\quad n(L \cap S) = 50$

$\because \quad n(L \cup S) = n(L) + n(S) - n(L \cap S)$

$\therefore \quad n(L \cup S) = 400 + 200 - 50$

$\Rightarrow \quad n(L \cup S) = 550$

But $\quad L \cup S \subseteq U$

$\Rightarrow \quad n(L \cup S) \le n(U)$

$\Rightarrow \quad 550 \le 500$, which is not true.

This is a contradiction.

So, the given data is incorrect.

EXAMPLE |9| If A and B are two sets containing 3 and 6 elements respectively, what can be the minimum number of elements in $A \cup B$? Also, find the maximum number of elements in $A \cup B$.

SOL Given $\quad n(A) = 3$

and $\quad n(B) = 6$

We know that,
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
Clearly, $n(A \cup B)$ will be maximum when $n(A \cap B)$ is minimum and it will be minimum when $n(A \cap B)$ is maximum. Therefore, two cases arise

***Case* I When $n(A \cap B)$ is minimum**

The minimum value of $n(A \cap B) = 0$

$\therefore \quad n(A \cup B) = n(A) + n(B) = 3 + 6 = 9$

Then, maximum value of $n(A \cup B)$ is 9.

***Case* II When $n(A \cap B)$ is maximum**

$n(A \cap B)$ will be maximum, if
$$A \subseteq B$$
In this case, $n(A \cap B) = 3$

$\therefore \quad n(A \cup B) = 3 + 6 - 3$
$$= 6$$
Thus, the minimum number of elements in $A \cup B$ is 6.

EXAMPLE |10| A survey shows that 63% of the Indian like cheese whereas 76% like apples. If x% of the Indian like both cheese and apples, then find the values of x.

SOL. Let the population of India be 100 crore.

Let C denotes the set of Indians who like cheese.

Then, $\quad n(C) = 63$ crore

Again, let A denotes the set of Indians who like apple.

Then, $\quad n(A) = 76$ crore

and $n(C \cap A) = x$ crore

Now, $n(C \cup A) = n(C) + n(A) - n(C \cap A)$

$\Rightarrow \quad n(C \cup A) = 63 + 76 - n(C \cap A)$

$\Rightarrow \quad n(C \cup A) + n(C \cap A) = 139$

$\Rightarrow \quad n(C \cap A) = 139 - n(C \cup A) \quad$ ...(i)

But $n(C \cup A) \le 100$

$\therefore \quad -n(C \cup A) \ge -100$

$\Rightarrow \quad 139 - n(C \cup A) \ge 139 - 100$

$\Rightarrow \quad 139 - n(C \cup A) \ge 39$

$\Rightarrow \quad n(C \cap A) \ge 39 \quad$ [using Eq. (i)] ...(ii)

Again, $C \cap A \subseteq C$ and $C \cap A \subseteq A$

$\Rightarrow n(C \cap A) \le n(C)$ and $n(C \cap A) \le n(A)$

$\Rightarrow n(C \cap A) \le 63$ and $n(C \cap A) \le 76$

$\Rightarrow \quad n(C \cap A) \le 63 \quad$ ...(iii)

From (ii) and (iii), we get
$$39 \le n(C \cap A) \le 63$$
$\Rightarrow \quad 39 \le x \le 63$

EXAMPLE |11| In a survey of 200 students of a school, it was found that 120 study Mathematics, 90 study Physics and 70 study Chemistry, 40 study Mathematics and Physics, 30 study Physics and Chemistry, 50 study Chemistry and Mathematics and 20 none of these subjects. Find the number of students who study all the three subjects. **[NCERT Exemplar]**

SOL. Let M, P and C respectively denote the set of students studying Mathematics, Physics and Chemistry and U denote the set of students who were participated in the survey. Then, we have,

$$n(U) = 200,\ n(M) = 120,\ n(P) = 90,\ n(C) = 70,$$
$$n(M \cap P) = 40,\ n(P \cap C) = 30,\ n(M \cap C) = 50$$

and $\quad n(M' \cap P' \cap C') = n(M \cup P \cup C)' = 20$

Now, $\quad n(M \cup P \cup C)' = n(U) - n(M \cup P \cup C)$

$\therefore \quad 20 = 200 - n(M \cup P \cup C)$

$\Rightarrow \quad n(M \cup P \cup C) = 200 - 20 = 180$

$\because \quad n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(P \cap C) - n(C \cap M) + n(C \cap M \cap P)$

$\therefore \quad 180 = 120 + 90 + 70 - 40 - 30 - 50 + n(C \cap M \cap P)$

$\Rightarrow \quad 180 = 280 - 120 + n(C \cap M \cap P)$

$\Rightarrow \quad 180 + 120 - 280 = n(P \cap C \cap M)$

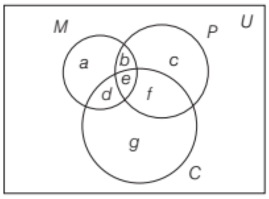
$\Rightarrow \quad n(P \cap C \cap M) = 300 - 280 = 20$

Hence, 20 students study all the three subjects.

EXAMPLE |12| In a survey of 25 students, it was found that 15 had taken Maths, 12 had taken Physics, 11 had taken Chemistry, 5 had taken Maths and Chemistry, 9 had taken Maths and Physics, 4 had taken Physics and Chemistry and 3 had taken all the three subjects. Find the number of students that had taken

(i) only Chemistry. (ii) only Maths.
(iii) only Physics.
(iv) Physics and Chemistry but not Maths.
(v) Maths and Physics but not Chemistry.
(vi) only one of the subjects.
(vii) atleast one of the three subjects.
(viii) None of the subjects.

SOL. Let M, P and C respectively denote the set of students who had taken Maths, Physics and Chemistry and denote the set of students who were participated the survey. Now, let in Venn diagram, a, b, c, d, e, f and g denote the number of students in respective regions.



From the Venn diagram, we get

$$n(M) = a + b + d + e,$$
$$n(P) = b + c + e + f,$$
$$n(C) = d + e + f + g,$$
$$n(M \cap P) = b + e,$$
$$n(P \cap C) = e + f,$$
$$n(M \cap C) = d + e \text{ and } n(M \cap P \cap C) = e,$$

Also, we have the followings number of students who had taken all three subjects is 3,

$$n(M \cap P \cap C) = 3 \Rightarrow e = 3$$

Also, we have the following:

Number of students who had taken Maths and Physics is 9, $\quad n(M \cap P) = 9 \Rightarrow b + e = 9$

$\Rightarrow \quad b + 3 = 9 \quad [\because e = 3]$

$\Rightarrow \quad b = 6$

Number of students who had taken Physics and Chemistry is 4

$$n(P \cap C) = 4 \Rightarrow e + f = 4$$

$\Rightarrow \quad 3 + f = 4 \quad [\because e = 3]$

$\Rightarrow \quad f = 1$

Number of students who had taken Maths and Chemistry is 5,

$$n(M \cap C) = 5 \Rightarrow d + e = 5$$

$\Rightarrow \quad d + 3 = 5 \quad [\because e = 3]$

$\Rightarrow \quad d = 2$

Number of students who had taken Maths is 15,

$$n(M) = 15$$

$\Rightarrow \quad a + b + d + e = 15$

$\Rightarrow \quad a + 6 + 2 + 3 = 15 \quad [\because b = 6, d = 2, e = 3]$

$\Rightarrow \quad a = 15 - 11 \Rightarrow a = 4$

Number of students who had taken Physics is 12,

$$n(P) = 12$$

$\Rightarrow \quad b + c + e + f = 12$

$\Rightarrow \quad 6 + c + 3 + 1 = 12 \quad [\because b = 6, e = 3, f = 1]$

$\Rightarrow \quad c = 12 - 10 \Rightarrow c = 2$

Number of students who had taken Chemistry is 11,

$$n(C) = 11 \Rightarrow d + e + f + g = 11$$

$\Rightarrow \quad 2 + 3 + 1 + g = 11 \quad [\because d = 2, e = 3, f = 1]$

$\Rightarrow \quad g = 11 - 6 \Rightarrow g = 5$

Now, *the number of students in various cases are as follows*

(i) Only Chemistry, $g = 5$ (ii) Only Maths, $a = 4$
(iii) Only Physics, $c = 2$
(iv) Physics and Chemistry but not Maths, $f = 1$
(v) Maths and Physics but not Chemistry, $b = 6$
(vi) Only one of the subjects, $a + c + g = 4 + 2 + 5 = 11$
(vii) Atleast one of the three subjects

$$a + b + c + d + e + f + g$$
$$= 4 + 6 + 2 + 2 + 3 + 1 + 5 = 23$$

(viii) None of the subjects,

$$25 - (a + b + c + d + e + f + g)$$
$$= 25 - 23 = 2$$

By Formula

Given, $n(U) = 25$, $n(M) = 15$, $n(P) = 12$, $n(C) = 11$, $n(M \cap C) = 5$, $n(M \cap P) = 9$, $n(P \cap C) = 4$, $n(M \cap P \cap C) = 3$

(i) Number of students that had taken Chemistry only

$= n(M' \cap P' \cap C) = n((M \cup P)' \cap C)$

$= n(C) - n((M \cup P) \cap C)$ $[\because n(A \cap B') = n(A) - n(A \cap B)]$

$= n(C) - n[(M \cap C) \cup (P \cap C)]$

$= n(C) - [n(M \cap C) + n(P \cap C) - n(M \cap P \cap C)]$

[using, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

and $n(M \cap C \cap P \cap C) = n\{M \cap P \cap (C \cap C)\}$
$= n(M \cap P \cap C)]$

$= 11 - \{5 + 4 - 3\} = 11 - 6 = 5$

(ii) Number of students that had taken Maths only

$= n(M \cap P' \cap C') = n\{M \cap (P \cup C)'\}$

$= n(M) - n[M \cap (P \cup C)]$

$= n(M) - n[(M \cap P) \cup (M \cap C)]$

$= n(M) - [n(M \cap P) + n(M \cap C) - n(M \cap P \cap C)]$

$= 15 - (9 + 5 - 3) = 15 - 11 = 4$

(iii) Number of students that had taken Physics only

$= n(P \cap M' \cap C') = n(P \cap (M \cup C)')$

$= n(P) - n(P \cap (M \cup C))$

$= n(P) - n[(P \cap M) \cup (P \cap C)]$

$= n(P) - [n(P \cap M) + n(P \cap C) - n(P \cap M \cap C)]$

$= 12 - (9 + 4 - 3) = 12 - 10 = 2$

(iv) Number of students that had taken Physics and Chemistry but not Maths

$= n(P \cap C \cap M')$

$= n(P \cap C) - n(P \cap C \cap M)$

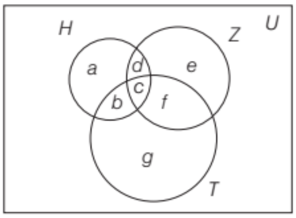
EXAMPLE |13| In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper Z, 9 read both H and Z, 11 read both H and T, 8 read both T and Z and 3 read all three newspapers. Find

(i) the number of people who read atleast one of the newspaper.

(ii) the number of people who read exactly one newspaper.

SOL. Let H, T and Z respectively denote the set of peoples who read newspapers H, T and Z and U denote the set of people who were participated in the survey.

Now, let in Venn diagram, a, b, c, d, e, f and g denote the number of peoples in respective regions.



Here, $n(U) = 60$ ...(i)

$n(H) = a + b + c + d = 25$ ...(ii)

$n(T) = b + c + f + g = 26$ ...(iii)

$n(Z) = c + d + e + f = 26$ ...(iv)

$n(H \cap Z) = c + d = 9$ ...(v)

$n(H \cap T) = b + c = 11$ ...(vi)

$n(T \cap Z) = c + f = 8$ ...(vii)

and $n(H \cap T \cap Z) = c = 3$ ...(viii)

On putting the value of c in Eq. (vii), we get

(v) Number of students that had taken Maths and Physics but not Chemistry

$= n(M \cap P \cap C')$

$= n(M \cap P) - n(M \cap P \cap C)$

$= 9 - 3 = 6$

(vi) Number of students that had taken only one of the subjects $= n(M) + n(P) + n(C)$

$- 2[n(M \cap P) + n(P \cap C) + n(M \cap C)]$
$+ 3n(P \cap M \cap C)$

$= 15 + 12 + 11 - 2\{9 + 4 + 5\} + 3 \times 3$

$= 38 - 36 + 9 = 11$

(vii) Number of students that had taken atleast one of the three subjects $= n(M \cup P \cup C)$

$= n(M) + n(P) + n(C) - n(M \cap P)$
$- n(P \cap C) - n(M \cap C) + n(M \cap P \cap C)$

$= 15 + 12 + 11 - 9 - 4 - 5 + 3 = 23$

(viii) Number of students that had taken none of the subjects $= n(M' \cap P' \cap C')$

$= n(M \cup P \cup C)'$

$= n(U) - n(M \cup P \cup C)$

$= 25 - 23 = 2$

$3 + f = 8 \Rightarrow f = 5$

On putting the value of c in Eq. (vi), we get

$3 + b = 11 \Rightarrow b = 8$

On putting the value of c in Eq. (v), we get

$3 + d = 9 \Rightarrow d = 6$

On putting the values of c, d and f in Eq. (iv), we get

$3 + 6 + e + 5 = 26 \Rightarrow e = 26 - 14 = 12$

On putting the values of b, c and f in Eq. (iii), we get

$8 + 3 + 5 + g = 26 \Rightarrow g = 26 - 16 = 10$

On putting the values of b, c and d in Eq. (ii), we get

$a + 8 + 3 + 6 = 25 \Rightarrow a = 25 - 17 = 8$

Now,

(i) Number of people who read atleast one of the three newspapers

$= n(H \cup T \cup Z)$

$= a + b + c + d + e + f + g$

$= 8 + 8 + 3 + 6 + 12 + 5 + 10$

$= 52$

(ii) Number of people who read exactly one newspaper

$= a + e + g = 8 + 12 + 10 = 30$

TOPIC PRACTICE 5

OBJECTIVE TYPE QUESTIONS

1 If X and Y are two sets such that $X \cup Y$ has 50 elements, X has 28 elements and Y has 32 elements, then number of elements $X \cap Y$ have
(a) 10 (b) 46
(c) 110 (d) 54

2 In a class of 35 students, 24 like to play cricket and 16 like to play football. Also, each student likes to play atleast one of the two games. The number of students who like to play both cricket and football, is
(a) 27 (b) 43
(c) 5 (d) 75

3 In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. The number of people who speak atleast one of these two languages, is
(a) 40 (b) 60
(c) 20 (d) 80

4 Each set X_r contains 5 elements and each set Y_r contains 2 elements and $\bigcup_{r=1}^{20} X_r = S = \bigcup_{r=1}^{n} Y_r$. If each element of S belongs to exactly 10 of the X_r's and to exactly 4 of the Y_r's, then n is equal to
(a) 10 (b) 20
(c) 100 (d) 50

5 Let A and B be two sets such that $n(A) = 0.16$, $n(B) = 0.14$ and $n(A \cup B) = 0.25$. Then, $n(A \cap B)$ is equal to
(a) 0.3
(b) 0.5
(c) 0.05
(d) None of the above

VERY SHORT ANSWER Type Questions

6 If A and B are two sets, such that $n(A) = 28$, $n(B) = 32$ and $n(A \cap B) = 10$, then find the value of $n(A \cup B)$.

7 If X and Y are two sets such that $n(X) = 28$, $n(Y) = 32$ and $n(X \cup Y) = 45$, then find $n(X \cap Y)$. [NCERT]

SHORT ANSWER Type I Questions

8 In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many can speak both Hindi and English? [NCERT]

9 Let $n(U) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$. Find $n(A' \cap B')$.

10 Let F_1 be the set of parallelograms, F_2 be the set of rectangles, F_3 be the set of rhombus and F_4 be the set of squares. Then, show that F_1 is equal to union of all sets. [NCERT Exemplar]

11 In a town with a population of 5000, 3200 people are egg-eaters, 2500 meat-eaters and 1500 eat both egg and meat. How many are pure vegetarians?

12 In a school, there are 20 teachers who teach Maths or Physics. Out of these, 12 teach Maths and 4 teach Physics and Maths. How many teach Physics?

13 In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many people speak atleast one of these two languages?

14 The members of a group of 400 people speak either Hindi or English or both. If 270 speak Hindi only and 50 speak both Hindi and English, then how many of them speak English only?

SHORT ANSWER Type II Questions

15 In a group of 70 people, 37 like coffee, 52 like tea and each person like atleast one of the two drinks. How many people like both coffee and tea? [NCERT]

16. In a survey of 400 movie viewers, 150 were listed as liking 'Veer Zaara', 100 were listed as liking 'Aitraaz' and 75 were listed as both liking 'Aitraaz' as well as 'Veer Zaara'. Find how many people were liking neither 'Aitraaz' nor 'Veer Zaara'?

17 In a survey of 400 students in a school, 100 were listed as taking apple juice, 150 as taking orange juice and 75 were listed as taking both apple as well as orange juice. Find how many sutdents were taking neither apple juice nor orange juice.

18 Out of 100 students, 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English and Mathematics, 7 in Mathematics and Science, 4 in English and Science, 4 in all the three. Find how many students passed in

(i) English and Mathematics but not in Science?

(ii) Mathematics and Science but not in English?

19 In a group of 65 people, 40 like Cricket, 10 like both Cricket and Tennis. How many like tennis only and not Cricket? How many like Tennis? **[NCERT]**

20 There are 200 individuals with a skin disorder, 120 has been exposed to chemical C_1, 50 to chemical C_2 and 30 to both the chemicals C_1 and C_2. Find the number of individuals exposed to

(i) chemical C_1 or chemical C_2.

(ii) chemical C_1 but not chemical C_2.

(iii) chemical C_2 but not chemical C_1.

LONG ANSWER Type Questions

21 In a group of 100 people, 65 like to play Cricket, 40 like to play Tennis and 55 like to play Volleyball. All of them like to play atleast one of the three games. If 25 like to play both Cricket and Tennis, 24 like to play both Tennis and Volleyball and 22 like to play both Cricket and Volleyball, then

(i) how many like to play all the three games?

(ii) how many like to play Cricket only?

(iii) how many like to play Tennis only?

Represent above information in a Venn diagram.

22 In a class of 140 students, 60 play Football, 48 play Hockey and 75 play Cricket, 30 play Hockey and Cricket, 18 play Football and Cricket, 42 play Football and Hockey and 8 play all the three games. Use Venn diagram to find

(i) students who do not play any of these three games.

(ii) students who play only Cricket.

(iii) students who play Football and Hockey, but not Cricket.

23 A college awarded 38 medals in Football, 15 in Basketball and 20 in Cricket. If these medals went to a total of 58 men and only three men got medals in all the three sports, then how many received medals in exactly two of the three sports? **[NCERT]**

24 From 50 students taking examinations in Mathematics, Physics and Chemistry, each of the student has passed in atleast one of the subject, 37 passed in Mathematics, 24 in Physics and 43 in Chemistry. Atmost 19 passed in Mathematics and Physics, atmost 29 in Mathematics and Chemistry and atmost 20 in Physics and Chemistry. Find the largest possible number that could have passed all three examinations. **[NCERT Exemplar]**

25. In a survey, it is found that 21 people like product A, 26 people like product B and 29 people like product C. If 14 people like products A and B, 12 people like products C and A, 14 people like products B and C, 8 people like all the three products. Find how many like product C only? **[NCERT]**

HINTS & ANSWERS

1 (a) Given, $n(X \cup Y) = 50$, $n(X) = 28$, $n(Y) = 32$, $n(X \cap Y) = ?$

By the formula,

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$, we find that

$$n(X \cap Y) = n(X) + n(Y) - n(X \cup Y)$$
$$= 28 + 32 - 50 = 10$$

2 (c) Use formula, $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

3 (b) Let X denote the set of people who speak French and Y denote the set of people who speak Spanish.

Given, $n(X) = 50$, $n(Y) = 20$ and $n(X \cap Y) = 10$

$$\therefore \quad n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$
$$= 50 + 20 - 10 = 60$$

Thus, 60 people speak atleast one of these two languages.

4 (b) We have

$$n(X_r) = 5$$
$$\therefore \quad n\left(\bigcup_{r=1}^{20} X_r\right) = 20 \times 5 = 100$$
$$\Rightarrow \quad n(S) = 100$$

But each element of S belong to excetly 10 of X_r's

So, $\frac{100}{10} = 10$ are the number of distinct elements in S.

Also each element of S belong to exactly 4 of the Y_r s and Y_r contains 2 elements

$\therefore$ If S has n number of Y_r in it then

$$\frac{2n}{4} = 10$$
$$\Rightarrow \quad n = 20$$

5 (c) We have

$n(A) = 0.16, n(B) = 0.14$ and $n(A \cup B) = 0.25$

We know,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$\therefore \quad 0.25 = 0.16 + 0.14 - n(A \cap B)$$
$$\Rightarrow \quad n(A \cap B) = 0.05$$

6 $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 28 + 32 - 10 = 50$

7 Use the formula, $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

Ans. 15

8 $n(H) = 250$, $n(E) = 200$ and $n(H \cup E) = 400$

$\because n(H \cup E) = n(H) + n(E) - n(H \cap E)$

Ans. 50

9 $n(A' \cap B') = n(U) - n(A \cup B)$ **Ans.** 300

10 All Rectangles, Rhombus and Square are parallelogram because its opposite sides are equal and parallel. Therefore $F_2 \subset F_1$, $F_3 \subset F_1$ and $F_4 \subset F_1$

$\Rightarrow \quad F_1 = F_2 \cup F_3 \cup F_4$

11. Let E be the set of people who are egg-eaters M be set of people who are meat-eaters and U be the set of people in the town.

Then, number of pure vegetarians

$= n(U) -$ Number of non- vegetarians

$= n(U) - n(E \cup M)$ **Ans.** 800

12. $n(M \cup P) = 20$, $n(M) = 12$, $n(P \cap M) = 4$

$\because n(M \cup P) = n(M) + n(P) - n(M \cap P)$ **Ans.** 12

13. $n(F \cup S) = 50 + 20 - 10$ **Ans.** 60

14. $n(A \cup B) = n(\text{only } A) + n(\text{only } B) + n(A \cap B)$

$\Rightarrow n(\text{only } B) = 400 - 270 - 50$ **Ans.** 80

(i) Required number of individuals $= n(C_1 \cup C_2)$

$= n(C_1) + n(C_2) - n(C_1 \cap C_2)$

Ans. 140

(ii) Required number of individuals $= n(C_1 \cap C_2')$

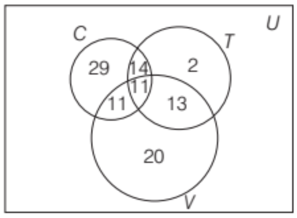
$= n(C_1) - n(C_1 \cap C_2) = 120 - 30 = 90$

(iii) Required number of individuals $= n(C_1' \cap C_2)$

$= n(C_2) - n(C_1 \cap C_2) = 50 - 30 = 20$

21. We have, $n(C) = 65$, $n(T) = 40$, $n(V) = 55$

$n(C \cup T \cup V) = 100$; $n(C \cap T) = 25$, $n(T \cap V) = 24$ and $n(C \cap V) = 22$.



Solve as Example 1.

(i) 11 (ii) 29 (iii) 2

22. Slve as Example 11 and 12

Ans. (i) 39 (ii) 35 (iii) 34

23. Solve as Example 12. **Ans.** 9

24. Let P, C and M respectively denote the set of students who were passed in physics, chemistry and mathematics. Then, we have $n(P \cup C \cup M) = 50$, $P(M) = 37$, $n(P) = 24$, $n(C) = 43$, $n(M \cap P) \le 19$, $n(M \cap C) \le 29$ and $n(P \cap C) \le 20$.

Clearly $n(P \cup C \cup M) = n(P) + n(C) - n(M) - n(P \cap C)$

$- n(C \cap M) - n(P \cap M) + n(P \cap C \cap M) \le 50$

15. 19

16. The number of people who were liking neither (A) 'Aitraaz' nor 'Veer Zaara' (V) is given by

$n(V' \cap A') = n(V \cup A)'$ [by De Morgan's law]

$= n(U) - n(V \cup A)$

$= n(U) - n(V) - n(A) + n(V \cap A)$

17. Solve as Example 4. **Ans.** 225

18. $n(E) = 15$, $n(M) = 12$, $n(S) = 8$, $n(E \cap M) = 6$

$n(M \cap S) = 7$, $n(E \cap S) = 4$, $n(E \cap M \cap S) = 4$

(i) $n(E \cap M \cap \overline{S}) = n(E \cap M) - n(E \cap M \cap S)$

$= 6 - 4 = 2$

(ii) $n(M \cap S \cap \overline{E}) = n(M \cap S) - n(E \cap M \cap S)$

$= 7 - 4 = 3$

19. $n(T \cup C) = n(T) + n(C) - n(T \cap C)$

$\therefore \quad n(T) = 35$

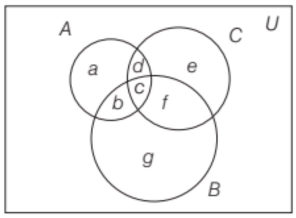
Now, $n(T \cap \overline{C}) = n(T) - n(T \cap C) = 35 - 10 = 25$

Ans. 25, 35

20. $n(U) = 200$; $n(C_1) = 120$, $n(C_2) = 50$; $n(C_1 \cap C_2) = 30$

Ans. 14

25. Let in Venn diagram, a, b, c, d, e, f and g denote the number of peoples in respective regions.



Then, $n(A) = a + b + c + d = 21$ ...(i)

$n(B) = b + c + f + g = 26$ ...(ii)

$n(C) = c + d + e + f = 29$ ...(iii)

$n(A \cap B) = b + c = 14$ ...(iv)

$n(C \cap A) = c + d = 12$ ...(v)

$n(B \cap C) = c + f = 14$ ...(vi)

and $n(A \cap B \cap C) = c = 8$ ...(vii)

On solving, we get the number of people who like product C only, $e = 11$.

SUMMARY

- A well-defined collection of objects, is called **set**.
- A set, which is empty or consists of a definite number of elements, is called **a finite set**.
- A set which consists of infinite number of elements, is called an **infinite set**.
- A set which does not contain any element, is called an **empty set** or **null** or **void set**.
- A set, consisting of a single element, is called a **singleton set**.
- If every element of A is an element of B, then A is called a **subset** of B and written as $A \subseteq B$. Also, set B is called **superset** of A.
- A set which is superset of each one of the given sets, is called **universal set**.
- The **union** of sets A and B is the set of all those elements which belong to either in A or in B or in both. It is denoted by $A \cup B$.
  $\therefore$ $A \cup B = \{x : x \in A \text{ or } x \in B\}$
- The **intersection** of sets A and B is the set of all those elements which belong to both A and B. It is denoted by $A \cap B$.
  $\therefore$ $A \cap B = \{x : x \in A \text{ and } x \in B\}$
- The **difference** of two sets A and B in this order is the set of all those elements of A which do not belong to B. It is denoted by $A–B$.
  $\therefore$ $A - B = \{x : x \in A \text{ and } x \notin B\}$
- The **symmetric difference** of A and B is the set $(A - B) \cup (B - A)$. It is denoted by $A \Delta B$.
  $\therefore$ $A \Delta B = (A - B) \cup (B - A)$ but $x = \{x : x \in A \text{ or } x \in B \notin A \cap B\}$.
- Two sets A and B are said to be **disjoint sets**, if they have no common element i.e. $A \cap B = \phi$.
- Let U be the universal set and A be any subset of U then **complement** of A with respect to U is the set of all those elements of U which are not in A. It is denoted by $\bar{A}$ or A'.
  Thus, $A' = \{x : x \in U \text{ and } x \notin A\}$
- **Important Results**
  (i) $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$
  (ii) $n(A) = n(A - B) + n(A \cap B)$
  (iii) $n(B) = n(B - A) + n(A \cap B)$
  (iv) $n(A \Delta B) = n[(A - B) \cup (B - A)] = n(A - B) + n(B - A)$
  $= n(A) + n(B) - 2n(A \cap B)$ [since, $(A - B)$ and $(B - A)$ are disjoint sets]
  (v) $n(A' \cup B') = n[(A \cap B)'] = n(U) - n(A \cap B)$
  (vi) $n(A' \cap B') = n[(A \cup B)'] = n(U) - n(A \cup B)$
  (vii) $n(A - B) = n(A \cap B)' = n(A) - n(A \cap B)$
  (viii) If A, B and C are finite sets, then
  (a) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
  (b) $n(A \text{ only}) = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$
  (c) $n(\bar{A} \cap \bar{B} \cap \bar{C}) = n(U) - n(A \cup B \cup C)$

CHAPTER PRACTICE

OBJECTIVE TYPE QUESTIONS

1 Let $B = \left\{x : x = \frac{1}{2n-1}, 1 \le n \le 5, \text{ where } n \in N\right\}$,
then B equals

(a) $\left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{9}, \frac{1}{11}\right\}$
(b) $\left\{1, \frac{1}{3}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}\right\}$
(c) $\left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}\right\}$
(d) None of these

2 The set $(A \cap B')' \cup (B \cap C)$ is equal to **[NCERT Exemplar]**

(a) $A' \cup B \cup C$
(b) $A' \cup B$
(c) $A' \cup C'$
(d) $A' \cap B$

3 If A and B are two sets, $A \cap (A \cup B)$ equals to **[NCERT Exemplar]**

(a) A (b) B (c) ϕ (d) $A \cap B$

4 Two finite sets have m and n elements. The number of subsets of the first set is 112 more than of the second. The values of m and n are respectively

(a) 4, 7 (b) 7, 4 (c) 4, 4 (d) 7, 7

5 The intersection of all the intervals having the form $\left[1 + \frac{1}{n}, 6 - \frac{2}{n}\right]$, where n is a positive integer, is

(a) $[1, 6]$
(b) $(1, 6)$
(c) $[2, 4]$
(d) $\left[\frac{3}{2}, 5\right]$

6 In an office, every employee likes at least one of tea, coffee and milk. The number of employees who like only tea, only coffee, only milk and all the three are all equal. The number of employees who like only tea and coffee, only coffee and milk and only tea and milk are equal and each is equal to the number of employees who like all three. Then, a possible value of the number of employees in the office is

(a) 65 (b) 90 (c) 77 (d) 85

VERY SHORT ANSWER Type Questions

7 What is the difference between a collection and a set? Give reason to support your answer.

8 Match each of the sets on the left described in the roster form with the same set on the right described in the set-builder form.

(i) {H, R, A, Y, N}	(a) $\{x : x$ is a natural number and a divisor of 18$\}$
(ii) {L, I, T, E}	(b) $\{x : x$ is a letter of the word HARYANA$\}$
(iii) {1, 2, 3, 4, 6, 12}	(c) $\{x : x$ is a letter of the word LITTLE$\}$
(iv) {1, 2, 3, 6, 9, 18}	(d) $\{x : x$ is a natural number and is a divisor of 12$\}$

9 Find the number of subsets of the set $A = \{1, 4, 5\}$.

10 List all the proper subsets of the set $A = \{a, b\}$.

SHORT ANSWER Type I Questions

11 Which of the following sets are empty set, singleton set and equal set.

$A = \{x : 2x = 10 \text{ and } x^2 - 7x + 10 = 0\}$

$B = \{x : x^2 - 16x + 55 = 0 \text{ and } x^2 = 25\}$

$C = \left\{x : -\frac{1}{2} \le x \le \frac{1}{2}\right\}$, $D = \{x : 0 \le 4x^2 \le 1\}$

12 If $n(A \cup B) = 25$, $n(A) = 12$, $n(A - B) = 8$, then write the number of elements in $A \cap B$ and $B - A$.

13 If $U = \{a, b, c, d, e, f\}$, $A = \{a, b, c\}$, $B = \{c, d, e, f\}$, $C = \{c, d, e\}$ and $D = \{d, e, f\}$, then tabulate the following sets.

(i) $A \cap D$ (ii) $A \cap C$ (iii) $U \cap D$
(iv) $A \cup \phi$ (v) $(U \cap \phi)'$ (vi) $(U \cup A)'$.

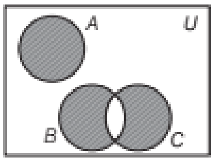
14 If $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 3, 5, 8\}$, $C = \{2, 5, 7, 8\}$, verify that $A - (B \cup C) = (A - B) \cap (A - C)$.

15 Let A and B be two sets such that $n(A - B) = 14 + x$, $n(B - A) = 3x$ and $n(A \cap B) = x$. Draw a Venn diagram to illustrate information and if $n(A) = n(B)$, then find the value of x.

16 Let A and B be two sets. Then, prove that $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$.

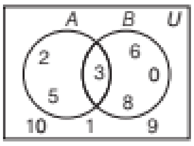
SHORT ANSWER Type II Questions

17 Express the shaded region of the following Venn diagram in terms of union and difference of sets A, B and C.



18 If $A = \{4^n - 3n - 1, n \in N\}$ and $B = \{9(n-1) : n \in N\}$, show that $A \subset B$.

19 From the adjoining Venn diagram, determine the following sets.



(i) $A \cup B$ (ii) $A \cap B$
(iii) $A - B$ (iv) $(A \cap B)'$

20 Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. Find the values of m and n.

21 If $n(A) = 4$ and $n(B) = 6$, then what can be the minimum number of elements in $A \cup B$?

22 State which of the following statements are true and which are false? Justify your answer.

(i) $35 \in \{x : x$ has exactly four positive factors$\}$

(ii) $128 \in \{y :$ the sum of all positive factors of y is $2y\}$

(iii) $3 \notin \{x : x^4 - 5x^3 + 2x^2 - 112x + 6 = 0\}$

(iv) $496 \in \{y :$ the sum of all the positive factors of y is $2y\}$

23 If $X = \{1, 2, 3\}$ and n represents any member of X, write the following sets containing all numbers represented by

(i) $4n$ (ii) $n + 6$
(iii) $\frac{n}{2}$ (iv) $n - 1$

24 Given set $A = \{1, 2, 3, 4, \ldots, 100\}$, write the subset

(i) X of A, whose elements are multiple of 7.
(ii) Y of A, whose elements are represented by $x + 3$, where $x \in A$.

25 Out of 600 car owners investigated, 500 owned Mahindra XUV and 200 owned TATA NANO, 50 owned both cars. Is this data correct?

LONG ANSWER Type Questions

26 A survey of 500 television viewers produced the following information, 285 watch football, 195 watch hockey, 115 watch basketball, 45 watch football and basketball 70 watch football and hockey, 50 watch hockey and basketball, 50 do not watch any of the three games. How many watch all the three games?

27 An investigator interviewed 100 students to determine their preferences for the three drinks: Milk (M), Coffee (C) and Tea (T). He reported that 10 students had all the three drinks M, C, T; 20 had M and C, 30 had C and T, 25 had M and T, 12 had M only, 5 had C only, 8 had T only. Using Venn diagram, find how many did not take any of the three drinks?

28 In a group of children, 35 play football out of which 20 play football only, 22 play hockey, 25 play cricket out of which 11 play cricket only. Out of these, 7 play cricket and football but not hockey, 3 play football and hockey but not cricket and 12 play football and cricket both? How many play all the three games? How many play cricket and hockey but not football? How many play hockey only? What is the total number of children in the group?

29 Of the members of three athletic teams in a certain school, 21 are in the basketball team, 26 in hockey team and 29 in the football team. 14 play hockey and basketball, 15 play hockey and football, 12 play football and basketball and 8 play all the three games. How many members are there in all?

CASE BASED Questions

30 The school organised a farewell party for 100 students and school management decided three types of drinks will be distributed in farewell party i.e. Milk (M), Coffee (C) and Tea (T). Organiser reported that 10 students had all the three drinks M,C,T. 20 students had M and C; 30 students had C and T; 25 students had M and T. 12 students had M only; 5 students had C only; 8 students had T only.

Based on the above information, answer the following questions.

(i) The number of students who did not take any drink, is
(a) 20 (b) 30 (c) 10 (d) 25

(ii) The number of students who prefer Milk is
(a) 47 (b) 45 (c) 53 (d) 50

(iii) The number of students who prefer Coffee is
(a) 47 (b) 53 (c) 45 (d) 50

(iv) The number of student who prefer Tea is
(a) 51 (b) 53 (c) 50 (d) 47

(v) The number of students who prefer Milk and Coffee but not tea is
(a) 12 (b) 10 (c) 15 (d) 20

31 In a library, 25 students read physics, chemistry and mathematics books. It was found that 15 students read mathematics, 12 students read physics while 11 students read chemistry.
5 students read both mathematics and chemistry,
9 students read physics and mathematics.
4 students read physics and chemistry and
3 students read all three subject books.

Based on the above information, answer the following questions.

(i) The number of student who reading only chemistry is
(a) 5 (b) 4 (c) 2 (d) 1

(ii) The number of students who reading only mathematics is
(a) 4 (b) 3 (c) 5 (d) 11

(iii) The number of students who reading only one of the subjects is
(a) 5 (b) 8 (c) 11 (d) 6

(iv) The number of student who reading at least one of the subject is
(a) 20 (b) 22 (c) 23 (d) 21

(v) The number of students who reading none of the subject is
(a) 2 (b) 4 (c) 3 (d) 5

32 In an University, out of 100 students 15 students offered Mathematics only, 12 students offered Statistics only, 8 students offered only Physics, 40 students offered Physics and Mathematics, 20 students offered Physics and Statistics, 10 students offered Mathematics and Statistics, 65 students offered Physics.

Based on the above information answer the following questions

(i) The number of students who offered all the three subjects.
(a) 4 (b) 3 (c) 2 (d) 5

(ii) The number of students who offered Mathematics.
(a) 62 (b) 65 (c) 55 (d) 60

(iii) The number of students who offered statistics
(a) 31 (b) 35 (c) 39 (d) 34

(iv) The number of students who offered mathematics and statistics but not physics.
(a) 7 (b) 6 (c) 5 (d) 4

(v) The number of students who did not offer any of the above three subjects.
(a) 4 (b) 1 (c) 5 (d) 3

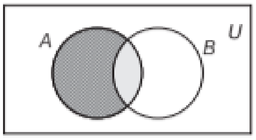
HINTS & ANSWERS

1. (c) Put $n = 1, 2, 3, 4, 5$ in $x = \frac{1}{2n-1}$

$\therefore \quad x = \frac{1}{1}, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}$

2. (b) We know that, $(A \cap B)' = A' \cup B'$ and $(A')' = A$

3. (a)



4. (b) **5.** (c) **6.** (c)

7. Every set is a collection but a collection is not necessarily a set. Only well-defined collection are sets e.g. Collection of most talented writers of India is a collection but it is not a set.

8. (i) $\to$ (b); (ii) $\to$ (c), (iii) $\to$ (d); (iv) $\to$ (a)

9. Numbr of subsets of $A = 2^3$ **Ans.** 8

10. $\phi, \{a\}, \{b\}$

11. $A = \{x : x = 5 \text{ and } (x-2)(x-5) = 0\} = \{5\}$

$B = \{x : (x-11)(x-5) = 0 \text{ and } x = \pm 5\} = \{5\}$

$C = \left\{x : -\frac{1}{2} \le x \le \frac{1}{2}\right\}$ and $D = \{x : 0 \le 4x^2 \le 1\}$

$= \left\{x : 0 \le x^2 \le \frac{1}{4}\right\} = \left\{x : -\frac{1}{2} \le x \le \frac{1}{2}\right\}$

$\therefore \quad C = D$

Ans. A and B are singleton sets, C and D are equal sets.

12. $n(A - B) = n(A) - n(A \cap B)$

$\Rightarrow n(A \cap B) = 12 - 8 = 4$

$\because \quad n(B) = n(A \cup B) + n(A \cap B) - n(A) = 25 + 4 - 12 = 17$

$\therefore \; n(B - A) = n(B) - n(B \cap A)$

Ans. $n(A \cap B) = 4, n(B - A) = 13$

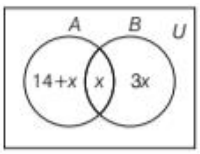
13. (i) ϕ (ii) $\{c\}$ (iii) $\{d, e, f\}$ (iv) $\{a, b, c\}$ (v) U (vi) ϕ

14. $B \cup C = \{1, 2, 3, 5, 7, 8\}$

$\therefore \; A - (B \cup C) = \{4\}$

and $(A - B) \cap (A - C) = \{2, 4\} \cap \{1, 3, 4\} = \{4\}$

15. $x = 7$



16. $A \cup (B \,\Delta\, C)$ or $A \cup \{(B - C) \cup (C - B)\}$

17. Now, $4^n - 3n - 1 = (3+1)^n - 3n - 1$

$= 1 + 3n + \frac{n(n-1)}{2} \cdot (3)^2 + \ldots - 3n - 1$

$= 9\left(\frac{n(n-1)}{2} + \ldots\right)$

and $\quad B = \{9(n-1) : n \in N\}$

19. (i) $\{0, 2, 3, 5, 6, 8\}$ (ii) $\{3\}$

(iii) $\{2, 5\}$ (iv) $\{0, 1, 2, 5, 6, 8, 9, 10\}$

20. $2^m = 56 + 2^n \Rightarrow 2^m - 2^n = 56$ **Ans.** $m = 6, n = 3$

21. $n(A \cup B) \ge n(B) = 6$

22. (i) $35 \in \{1, 5, 7, 35\}$

(ii) $128 \in \{1, 2, 4, 8, 16, 32, 64, 128\}$

Here, $y = \{1, 2, 4, 8, 16, 32, 64, 128\}$

Now, sum of elements of y

$= 1 + 2 + 4 + 8 + 16 + 32 + 64 + 138 = 255 \ne 2y$

(iii) We have, $x^4 - 5x^3 + 2x^2 - 112x + 6 = 0$

$\text{LHS} = (3)^4 - 5(3)^3 + 2(3)^2 - 112 \times 3 + 6$

$= 81 - 138 + 18 - 336 + 6 = -366 \ne 0$

(iv) $496 \in \{1, 2, 4, 8, 16, 62, 124, 248, 496\}$

Here, $\quad y = \{1, 2, 4, 8, 16, 62, 124, 248, 496\}$

Now, sum of elements of y

$= 1 + 2 + 4 + 8 + 16 + 62 + 124 + 248 + 496 \ne 2y$

(i) True (ii) False (iii) True (iv) False

23. (i) $\{4, 8, 12\}$ (ii) $\{7, 8, 9\}$ (iii) $\left\{\frac{1}{2}, 1, \frac{3}{2}\right\}$ (iv) $\{0, 1, 2\}$

24. (i) $\{7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98\}$

(ii) $\{4, 5, 6, \ldots, 100\}$

25. Let E and F denote the can owner owned XUV and TATA NANO, respectively.

Then, $\quad n(E) = 500, n(F) = 200,$

$n(E \cup F) = 600, n(E \cap F) = 50$

Now, $n(E \cup F) = 500 + 200 - 50 \Rightarrow 600 \ne 650$

Given data is incorrect.

26. $n(F) = 285, n(H) = 195, n(B) = 115,$

$n(F \cap B) = 45, n(F \cap H) = 70, n(H \cap B) = 50$

$n(\overline{A} \cap \overline{B} \cap \overline{C}) = 50, n(F \cup H \cup B) = 500$

$\because \quad n(F \cup H \cup B) = n(F) + n(H) + n(B)$

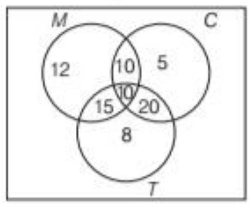
$- n(F \cap B) - n(F \cap H) - n(H \cap B) + n(F \cap H \cap B)$

Ans. 20

27. $n(U) = 100, n(M \cap C \cap T) = 10,$

$n(M \cap C) = 20, n(C \cap T) = 30, n(M \cap T) = 25,$

$n(M \text{ only}) = 12, n(C \text{ only}) = 5, n(T \text{ only}) = 8$



Now, $n(M \cup C \cup T) = 12 + 15 + 10 + 10 + 5 + 20 + 8 = 80$

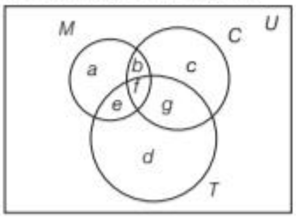
$\therefore \quad n(\overline{M} \cap \overline{C} \cap \overline{T}) = n(U) - n(M \cup C \cup T)$

$= 100 - 80 = 20$ **Ans.** 20

28. 5, 2, 12, 60

29. Similar as Example 13 (i). of Topic 5. **Ans.** 43

30. Consider the following Venn diagram



where,

$a =$ Number of students who had M only

$b =$ Number of students who had M and C Only

$c =$ Number of students who had C only

$d =$ Number of students who had T only

$e =$ Number of students who had M and T only

$f =$ Number of students who had three drinks M, C, T

and $g =$ Number of students who had C and T only

Then, we have

$$a = 12, b + f = 20, c = 5, d = 8, e + f = 25$$

$$f = 10 \text{ and } g + f = 30$$

$$\Rightarrow \quad a = 12, b = 10, c = 5, d = 8, e = 15, f = 10 \text{ and } g = 20$$

(i) (a) Number of students who did not take any drink

$$= 100 - (a + b + c + d + e + f + g)$$

$$= 100 - (12 + 10 + 5 + 8 + 15 + 10 + 20)$$

$$= 100 - 80 = 20$$

(ii) (a) Number of students who prefer Milk

$$= a + b + f + e = 12 + 10 + 10 + 15 = 47$$

(iii) (c) Number of students who prefer Coffee

$$= b + c + f + g = 10 + 5 + 10 + 20 = 45$$

(iv) (b) Number of students who prefer Tea

$$= d + e + f + g = 8 + 15 + 10 + 20 = 53$$

(v) (b) Number of students who prefer Milk and Coffee but not Tea is b, i.e. 10

31. Let M denotes set of student who reading mathematics books, P denotes who reading Physics books and C denotes who reading chemistry books.

We have,

$$n(U) = 25, n(M) = 15, n(P) = 12, n(C) = 11$$

$$n(M \cap C) = 5, n(M \cap P) = 9, n(P \cap C) = 4,$$

$$n(M \cap P \cap C) = 3$$

(i) (a) Required number of students

$$= n(M' \cap P' \cap C)$$

$$= n((M \cap P)' \cap C)$$

$$= n(M' \cap P' \cap C)$$

$$= n((M \cap P)' \cap C)$$

$$= n(C) - n((M \cup P) \cap C)$$

$$[\because n(A \cap B') = n(A) - n(A \cap B)]$$

$$= n(C) - n((M \cap C) \cup (P \cap C))$$

$$= n(C) - (n(M \cap C) + n(P \cap C) - n(M \cap P \cap C))$$

$$= 11 - (5 + 4 - 3) = 5$$

(ii) (a) Required number of students

$$= (M \cap P' \cap C')$$

$$= n(M \cap (P \cap C)')$$

$$= n(M) - n(M \cap (P \cup C))$$

$$= n(M) - n((M \cap P) \cup (M \cap C))$$

$$= n(M) - (n(M \cap P) + n(M \cap C) - n(M \cap P \cap C))$$

$$= 15 - (9 + 5 - 3) = 4$$

(iii) (c) Required number of students

$$= n(M) + n(P) + n(C) - 2$$

$$[n(M \cap P) + n(P \cap C) + n(M \cap C)] + 3n(M \cap P \cap C)]$$

$$= 15 + 12 + 11 - 2[9 + 4 + 5] + 3 \times 3$$

$$= 38 - 36 + 9 = 11$$

(iv) (c) Required number of students

$$= n(M \cup P \cup C)$$

$$= n(M) + n(P) + n(C) - n(M \cap P) - n(P \cap C) - n(M \cap C) + n(M \cap P \cap C)$$

$$= 15 + 12 + 11 - 9 - 4 - 5 + 3$$

$$= 41 - 18 = 23$$

(v) (a) Required number of students

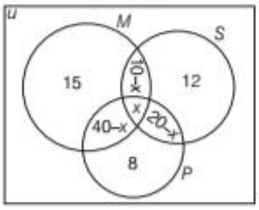
$$= n(M' \cap P' \cap C')$$

$$= n(M \cup P \cup C)'$$

$$= n(U) - n((M \cup P \cup C)$$

$$= 25 - 23 = 2$$

32. Let M, S and P be the sets of students wo offered Mathematics, Statistics and Physics respectively. Let x be the number of students who offered all the three subjects, then the number of members in different regions are shown in the following diagram.



From the Venn diagram, we get, the number of students who offered Physics.

$$= (40 - x) + x + (20 - x) + 8 = 65 \qquad \text{[given]}$$

$$\Rightarrow \quad 68 - x = 65 \Rightarrow x = 3.$$

(i) (b)

(ii) (a) The number of students who offered Mathematics

$$= 15 + (10 - x) + x + (40 - x)$$

$$= 65 - x = 65 - 3 = 62. \qquad [\because x = 3]$$

(iii) (c) The number of students who offered Statistics

$$= 12 + (10 - x) + x + (20 - x)$$

$$= 42 - x = 42 - 3 = 39. \qquad [\because x = 3]$$

$$= 12 + (10 - x) + x + (20 - x)$$

$$= 42 - x = 42 - 3 = 39. \qquad [\because x = 3]$$

(iv) (a) $10 - x = 10 - 3 = 7$

(v) (b) The number of students who offered anyone of the three subjects

$$= 15 + 12 + 8 + (10 - x) + (40 - x) + (20 - x) + x$$

$$= 105 - 2x$$

$$= 105 - 2 \times 3 = 99. \qquad [\because x = 3]$$

$\therefore$ The number of students who did not offer anyone of the three subjects $= 100 - 99 = 1$